

PRAVEEN

JEE 2026

Mathematics

Basic Maths

Lecture - 16

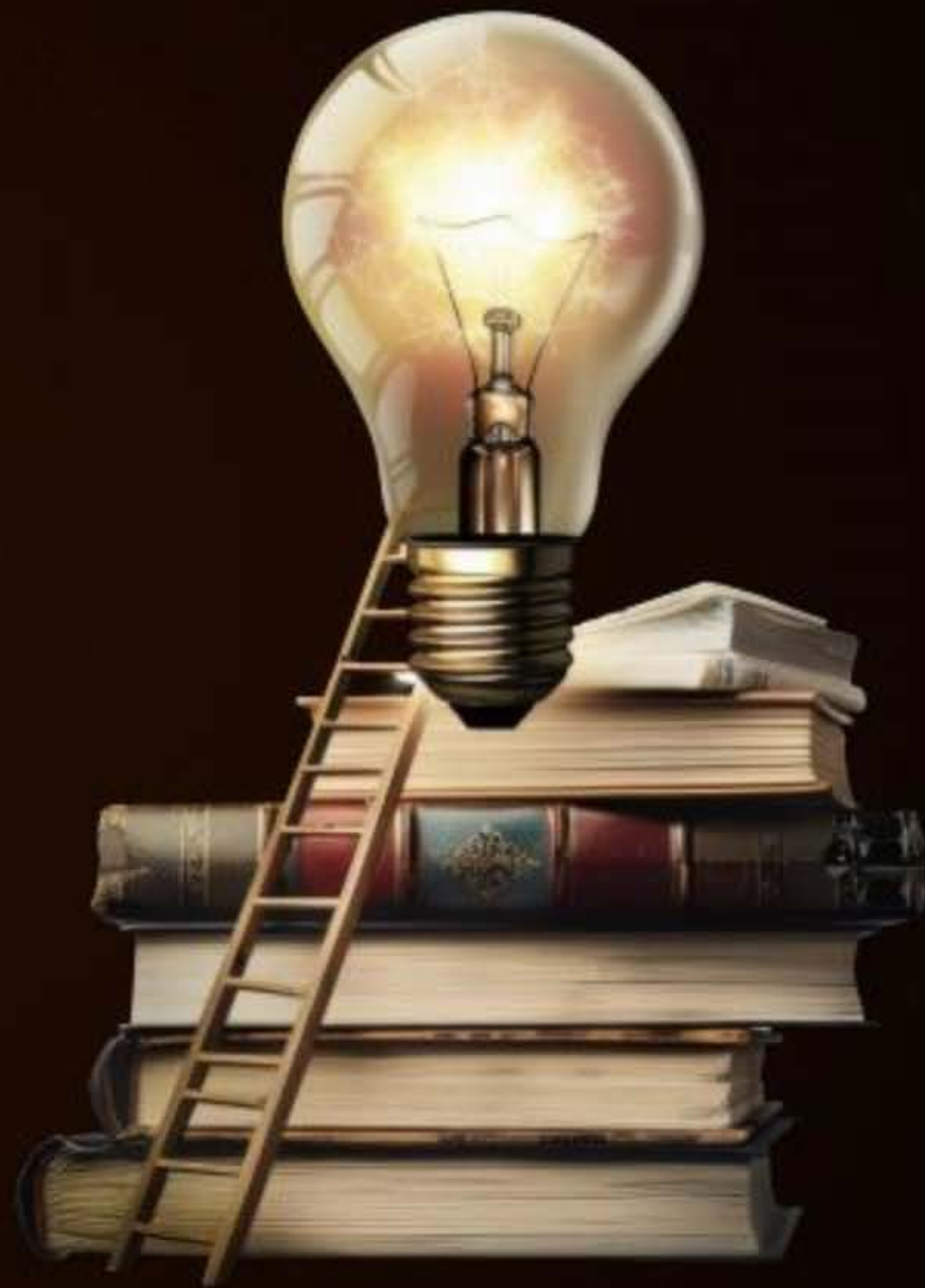
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Topics *to be covered*



- A** Graphs involving Modulus
- B** Irrational Inequalities
- C** Problem Practice



Homework Discussion

QUESTION

$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

$$\frac{1}{x-1} - \frac{1}{x-4} + \frac{4}{x-3} - \frac{4}{x-2} < \frac{1}{30}$$

$$\frac{x-4 - x+1}{(x-1)(x-4)} + \frac{4x-8 - 4x+12}{(x-2)(x-3)} < \frac{1}{30}$$

$$\frac{-3}{x^2-5x+4} + \frac{4}{x^2-5x+6} < \frac{1}{30} \quad \text{Let } x^2-5x=t$$

$$\frac{4}{t+6} - \frac{3}{t+4} < \frac{1}{30}$$

$$\frac{4t+16-3t-18}{(t+4)(t+6)} < \frac{1}{30}$$

$$\frac{(t-2)}{t^2+10t+24} - \frac{1}{30} < 0$$

(KTK 2)



$$\frac{30t-60 - t^2 - 10t - 24}{30(t+4)(t+6)} < 0$$

$$\frac{-t^2+20t-84}{(t+4)(t+6)} < 0$$

$$\frac{t^2-20t+84}{(t+4)(t+6)} > 0$$

$$\frac{(t-6)(t-14)}{(t+4)(t+6)} > 0$$

$$\frac{(x^2-5x-6)(x^2-5x-14)}{(x^2-5x+4)(x^2-5x+6)} > 0$$

$$\frac{(x-6)(x+1)(x-7)(x+2)}{(x-4)(x-1)(x-2)(x-3)} > 0$$

Ans. $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

- ☒ A no real solution
- ☒ B one prime solution
- ☐ C one integral solutions
- ☒ D no irrational solution

$$\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$$

$$\frac{\log_3 x}{\log_3 3\sqrt{x}} + \frac{\log_3 \sqrt{x}}{\log_3 3x} = 0$$

$$\frac{\log_3 x}{1 + \frac{1}{2} \log_3 x} + \frac{\frac{1}{2} \log_3 x}{1 + \log_3 x} = 0 \quad \text{let } \log_3 x = t$$

$$t \left(\frac{1}{1 + t/2} + \frac{1}{2(1+t)} \right) = 0$$

$$t = 0 \text{ or } \frac{2}{2+t} + \frac{1}{2(1+t)} = 0$$

$$x = 1$$

$$4 + 4t + 2 + t = 0$$

$$t = -6/5$$

$$x = 3^{-6/5} = \frac{1}{3^{6/5}} = \frac{1}{3^5 \sqrt[5]{3}}$$

$$3^{6/5} = 3^{\frac{5+1}{5}} = 3^{1 + \frac{1}{5}} = 3 \cdot 3^{1/5} = 3 \cdot \sqrt[5]{3}$$

Ans. A, B, D

**Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal**

Lecture = 15



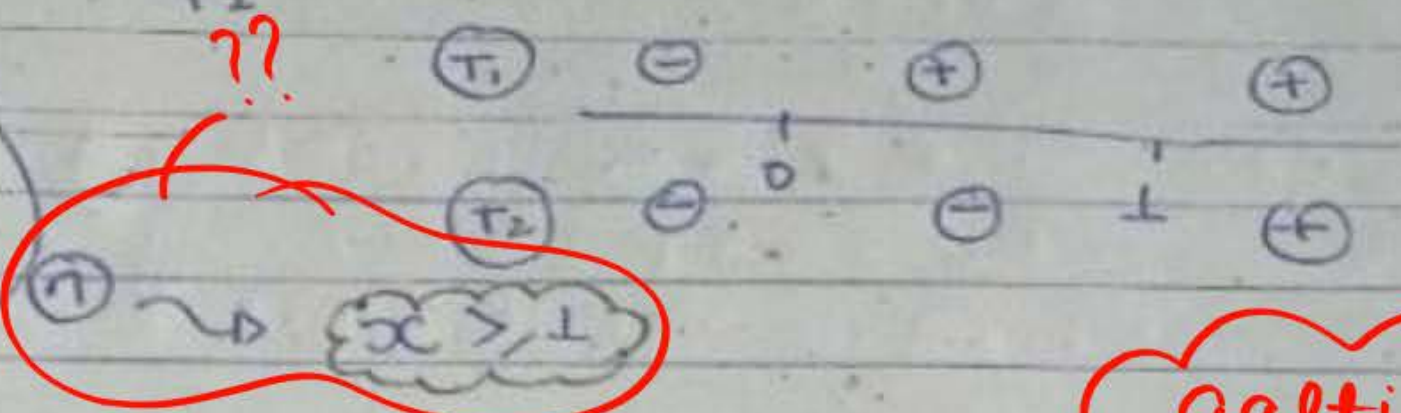
Tahol Solve $|x| + |x-1| \geq 7$

$\Rightarrow$ Case 01 $x \geq 1$

$\Rightarrow x + x - 1 \geq 7$

$\Rightarrow 2x - 8 \geq 0$

$\Rightarrow x - 4 \geq 0$



galti Batao

Case 02. $0 \leq x < 1$

$\Rightarrow x - x + 1 \geq 7$

$\Rightarrow 1 \geq 7$ (Not possible)

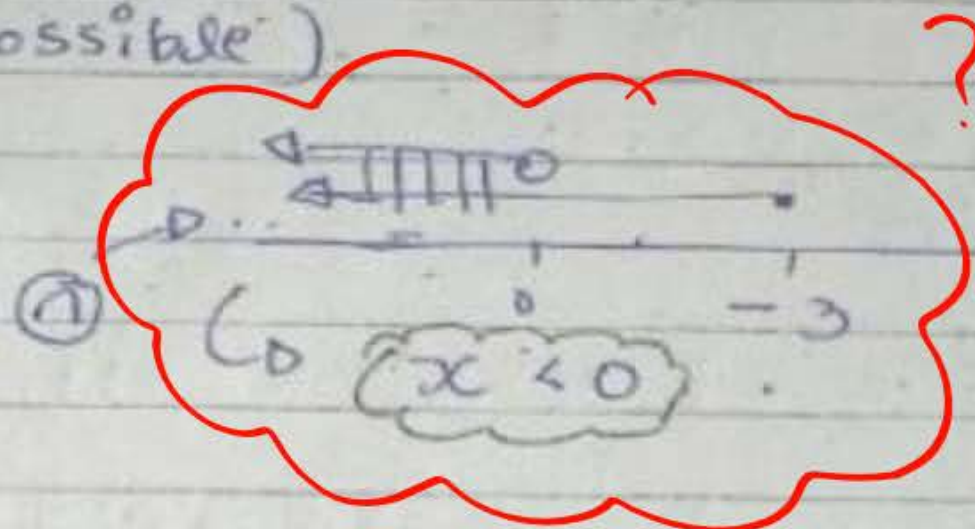
$\Rightarrow x \in \emptyset$

Case 03. $x < 0$

$\Rightarrow -x - x + 1 \geq 7$

$\Rightarrow -2x - 6 \geq 0$

$\Rightarrow x + 3 \leq 0$



$\Rightarrow$ Union of all cases

$x \in (-\infty, 0) \cup [1, \infty)$

Ans.

KTK-6

$$(A) \quad x^{\log_{10} x} = 100x$$

$$\log_{10} x^{\log_{10} x} = \log_{10} 100x$$

$$\log_{10} x \log_{10} x = \log_{10} 100x$$

$$(\log_{10} x)^2 = \log_{10} 100x$$

$$2 \log_{10} x = \log_{10} 100x \quad ??$$

$$\log_{10} x^2 = \log_{10} 100x$$

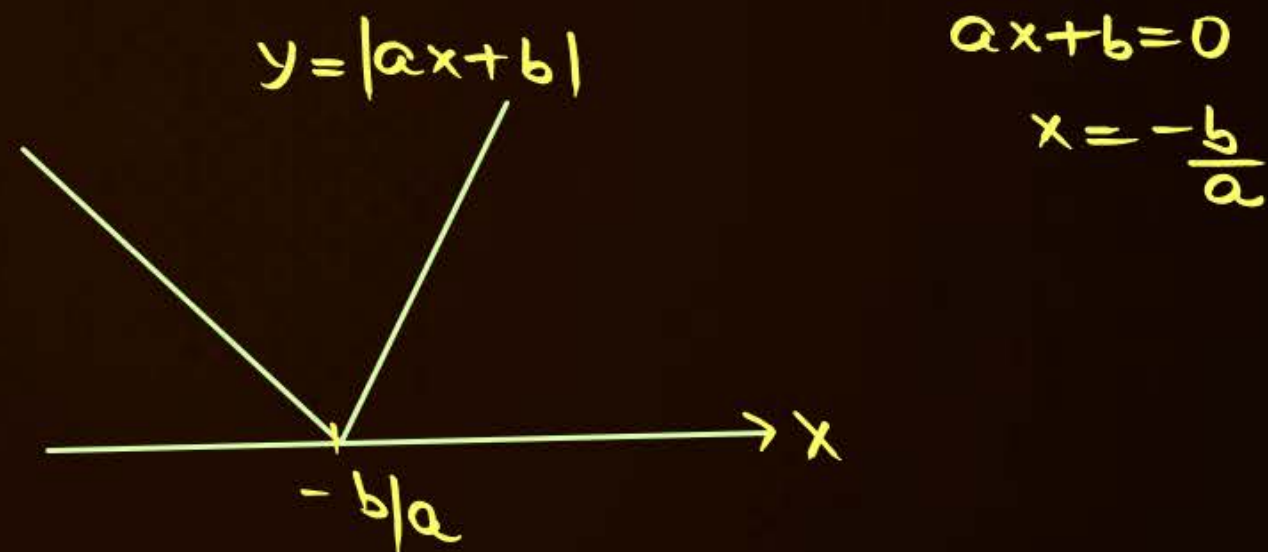
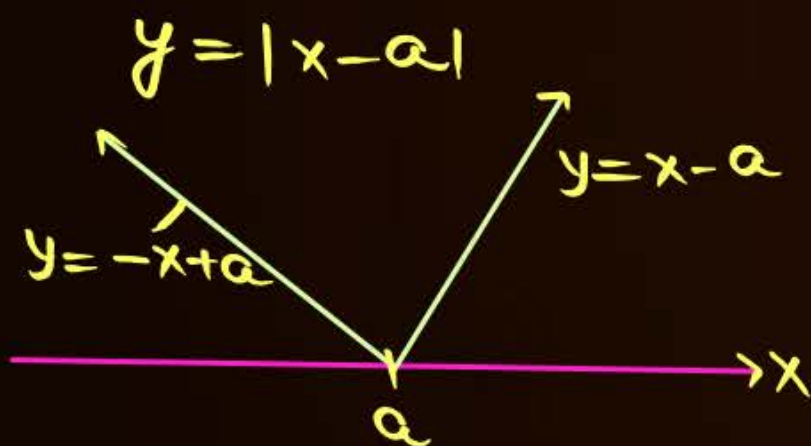
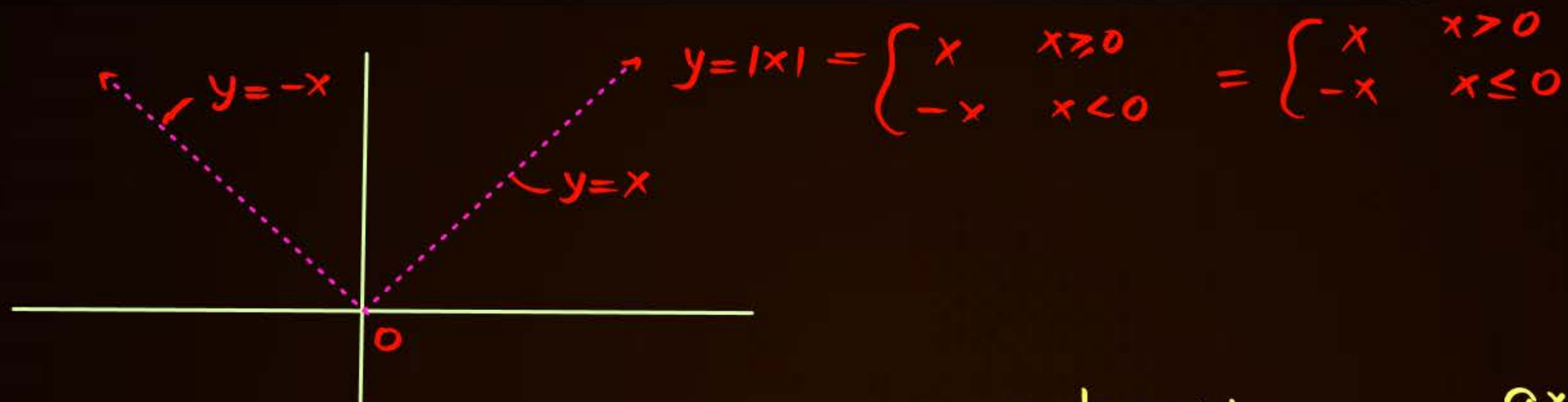
$$x^2 = 100x$$

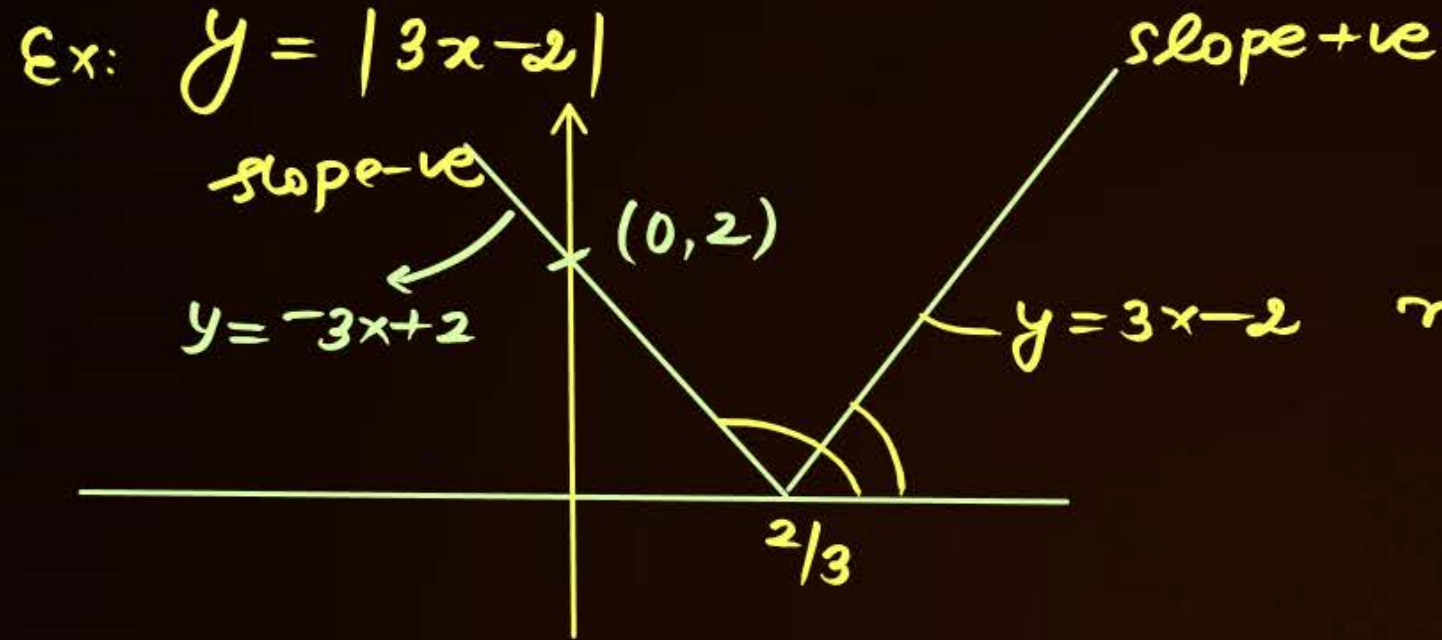
$$\boxed{x = 100}$$

→ rational
composite

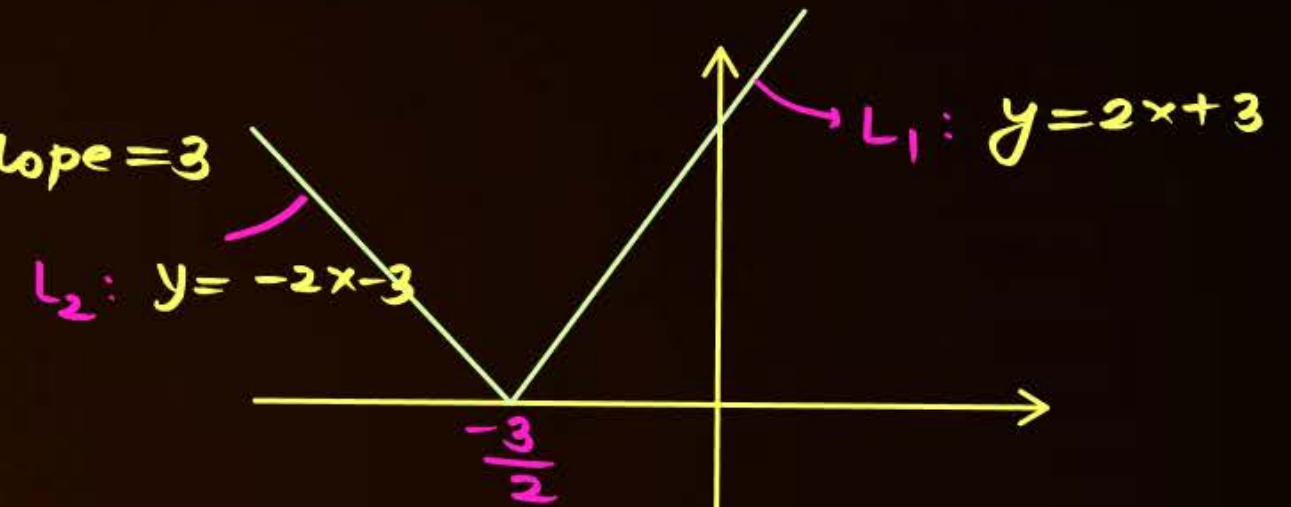


Problems Involving Graph of Modulus Function





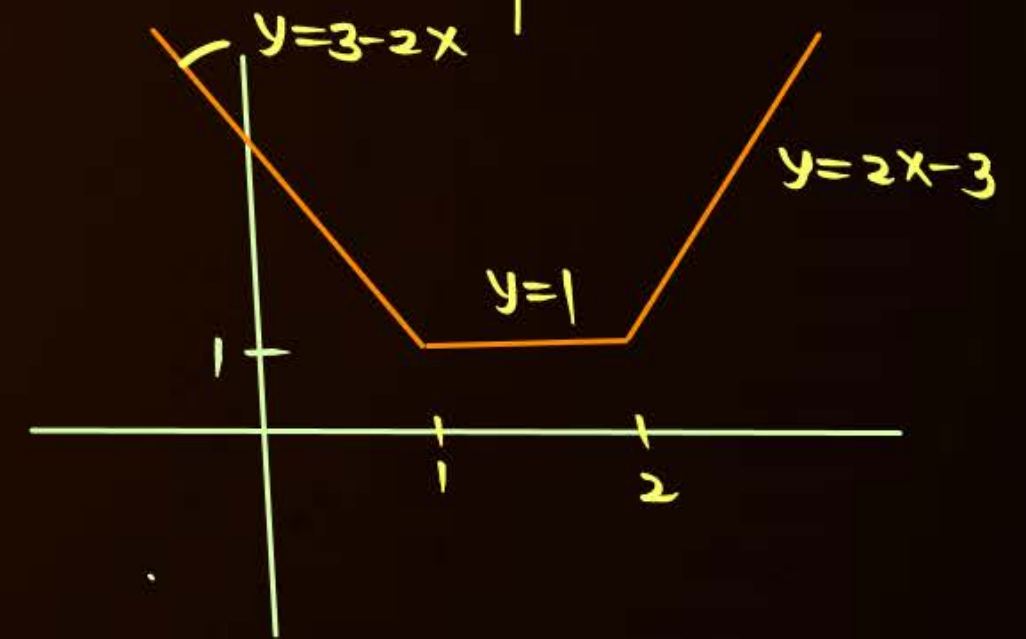
Ex: $y = |2x + 3|$



Ex: $y = |x - 1| + |x - 2|$

$\begin{array}{c} -ve & +ve & +ve \\ \hline -ve & 1 & -ve & 2 & +ve \end{array}$

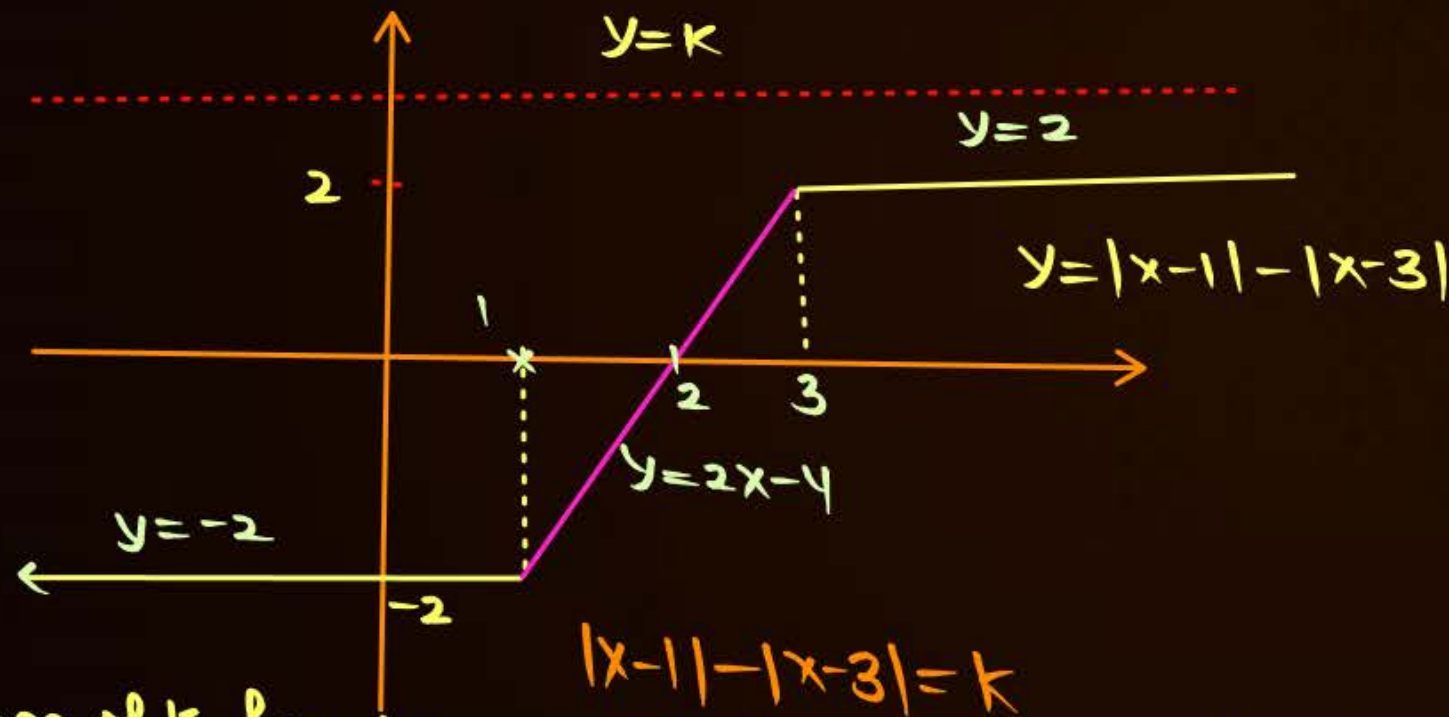
$$y = \begin{cases} -x + 1 - x + 2 & x \leq 1 \\ x - 1 + 2 - x & 1 < x < 2 \\ x - 1 + x - 2 & x \geq 2 \end{cases} = \begin{cases} 3 - 2x & x \leq 1 \\ 1 & 1 < x < 2 \\ 2x - 3 & x \geq 2 \end{cases}$$



$y=a$ is a line parallel to x axis

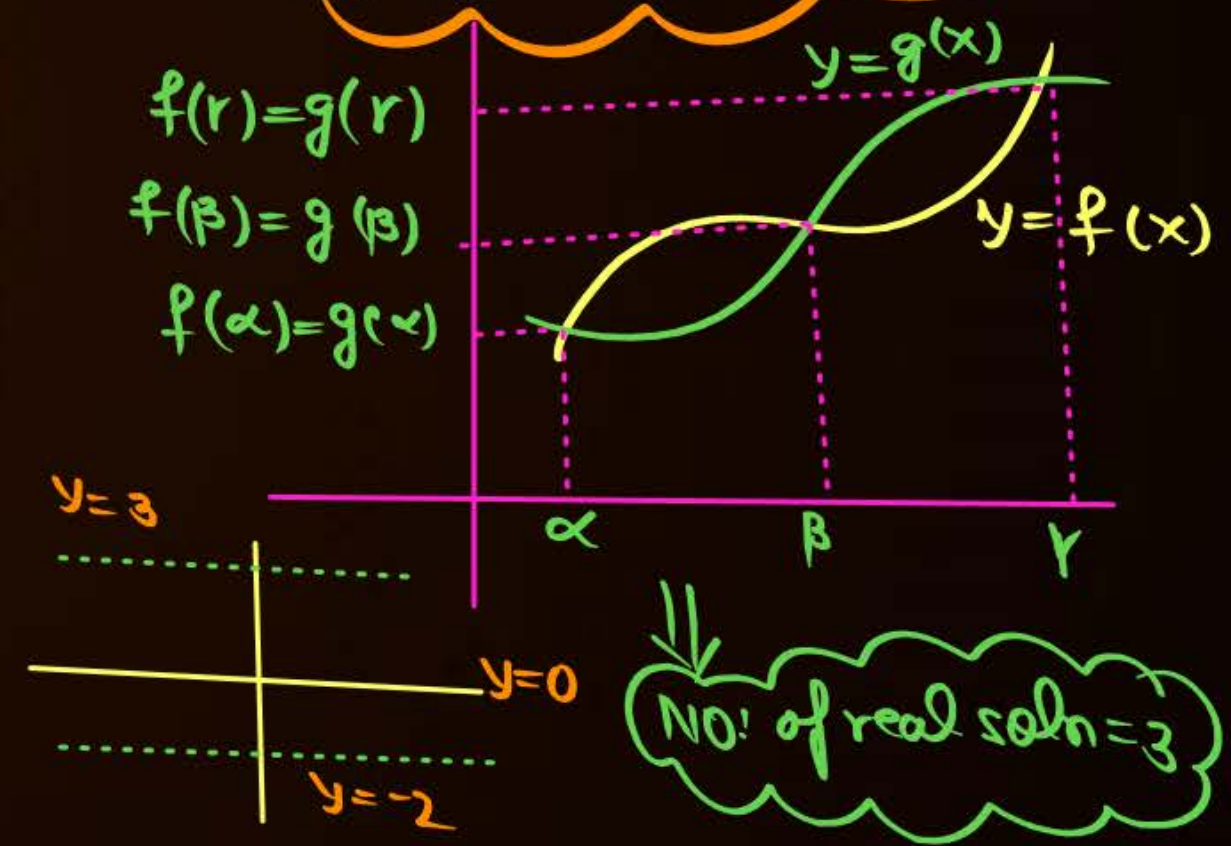
$$y = |x-1| - |x-3| \quad \begin{array}{c} - \quad + \quad + \\ -1 \quad - \quad 3 \end{array}$$

$$y = \begin{cases} -x+1+x-3 & x \leq 1 \\ x-1+x-3 & 1 < x < 3 \\ x-1-(x-3) & x \geq 3 \end{cases} = \begin{cases} -2 & x \leq 1 \\ 2x-4 & 1 < x < 3 \\ 2 & x \geq 3 \end{cases}$$



- Find Range of k for $|x-1| - |x-3| = k$
- (a) no real soln $\rightarrow k \in (-\infty, -2) \cup (2, \infty)$
 - (b) Exactly one real soln $\rightarrow k \in (-2, 2)$
 - (c) ∞ many soln $\rightarrow k = 2, -2$

No: of real solns of $f(x) = g(x)$ is obtained by no: of points of intersection of graphs $y = f(x)$ & $y = g(x)$



QUESTION



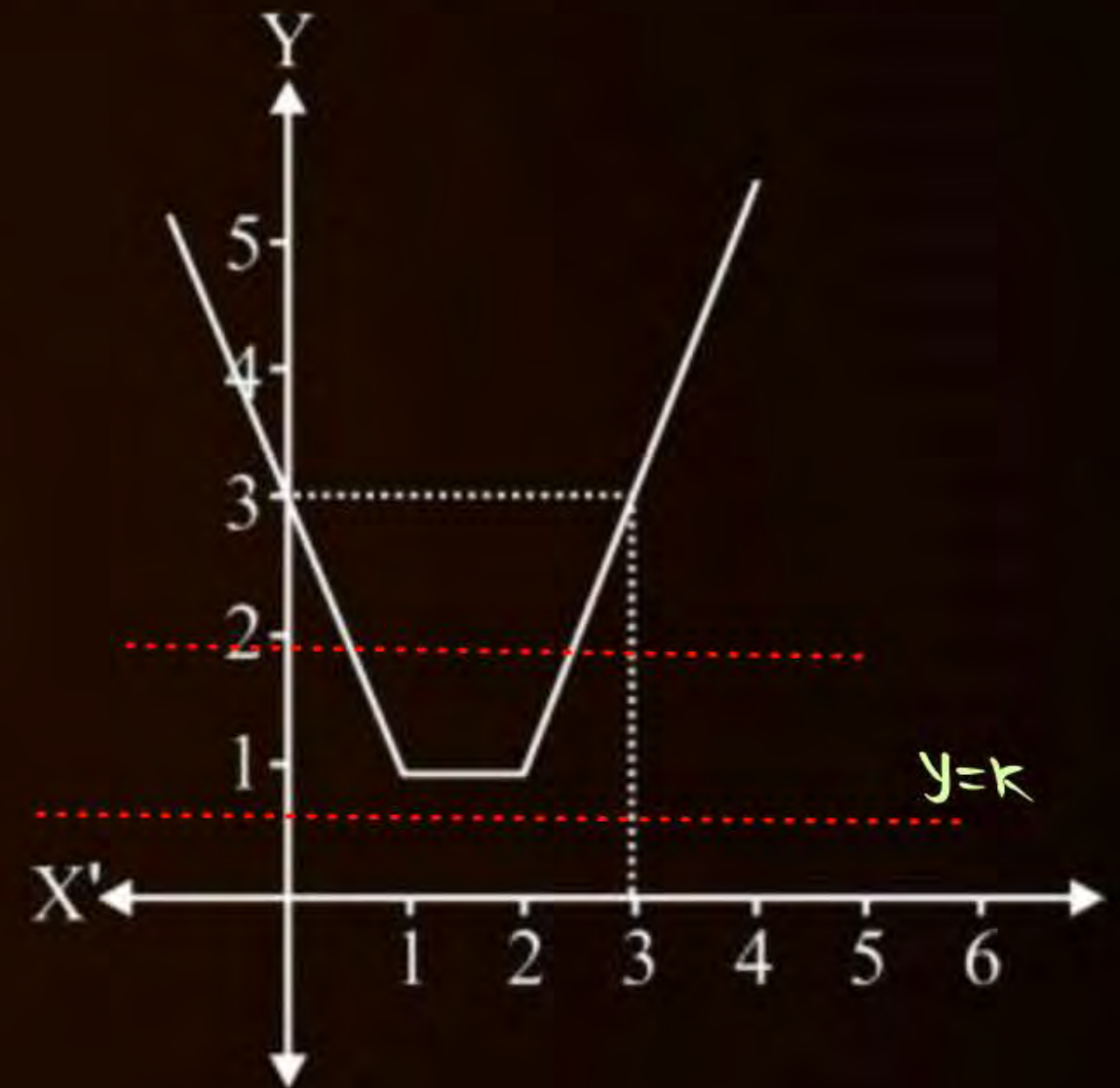
Draw graph of $y = |x - 2|$.

QUESTION



f $|x - 1| + |x - 2| = k$ then find range of k for which the equation has

- (a) No solution $\rightsquigarrow k \in (-\infty, 1)$
- (b) Two solution *only* $\rightsquigarrow k \in (1, \infty)$
- (c) Infinitely many solution $\rightsquigarrow k = 1$.



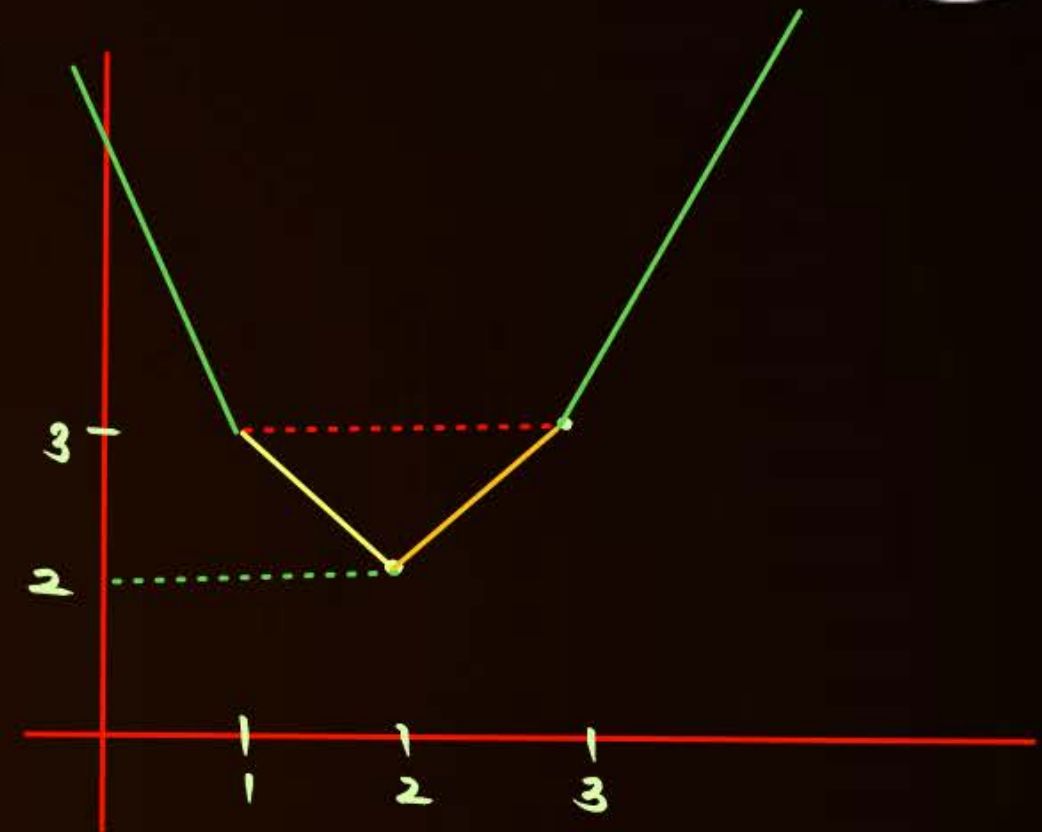
QUESTION



Draw graph of $y = \overset{T_1}{|x-1|} + \overset{T_2}{|x-2|} + \overset{T_3}{|x-3|}$.

$$y = \begin{cases} -x+1-x+2-x+3, & x \leq 1 \\ x-1-x+2-x+3, & 1 < x < 2 \\ x-1+x-2-x+3, & 2 \leq x \leq 3 \\ x-1+x-2+x-3, & x > 3 \end{cases}$$

$$= \begin{cases} 6-3x & x \leq 1 \\ 4-x & 1 < x < 2 \\ x & 2 \leq x \leq 3 \\ 3x-6 & x > 3 \end{cases}$$

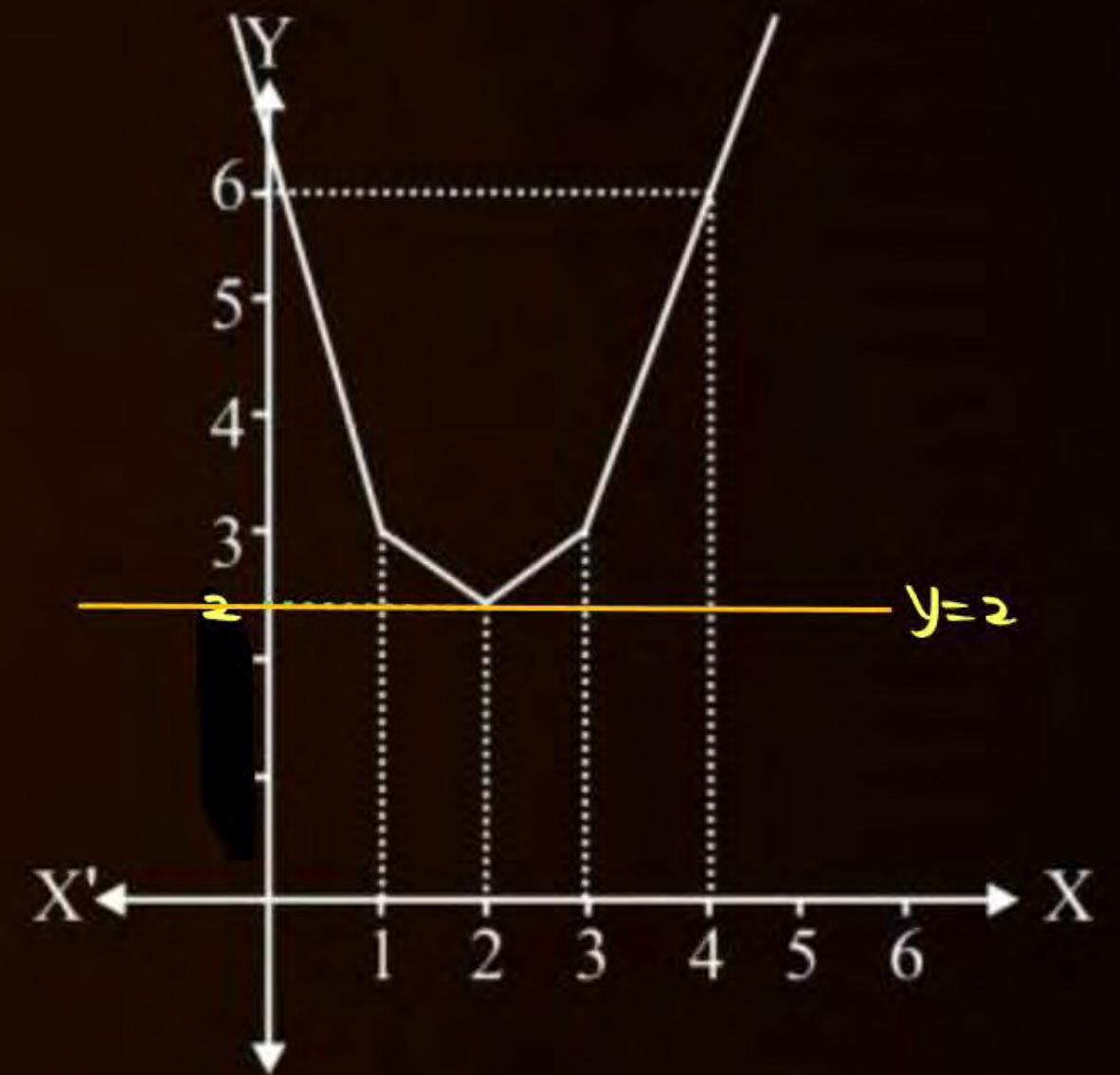


QUESTION



If $|x - 1| + |x - 2| + |x - 3| = k$ then find range of k for which the equation has

- (a) No solution $\leadsto k \in (-\infty, 2)$
- (b) Only one solution $\leadsto k = 2$
- (c) Two solution $\leadsto \underline{k \in (2, \infty)}$



QUESTION



Tahol

If $|x - 5| - |x - 7| = k$ then for

- A** $K = 2$ the equation has infinitely many solution
- B** $K = -2$ the equation has infinitely many solution
- C** $K \in (2, \infty)$ the equation has no solution
- D** $K \in (-2, 2)$ the equation has exactly one solution

QUESTION



Tah02

Draw graph of $y = |x| + |x - 2|$ and answer the following.

Find the range of k for which the equation $|x| + |x - 2| = k$ has

- A. No Solution
- B. Infinitely Many Solutions
- C. Only two solutions

QUESTION



Tah03

Draw the graph of $y = |x - 4| - |x + 3|$ and answer the following.

Find the range of k for which the equation $|x - 4| - |x + 3| = k$ has

- A. No Solution
- B. Infinitely Many Solutions
- C. Only two solutions

QUESTION



Tanoy

Consider the function $f(x) = |x - 1| - 2|x + 2| + |x + 3|$

Column-I

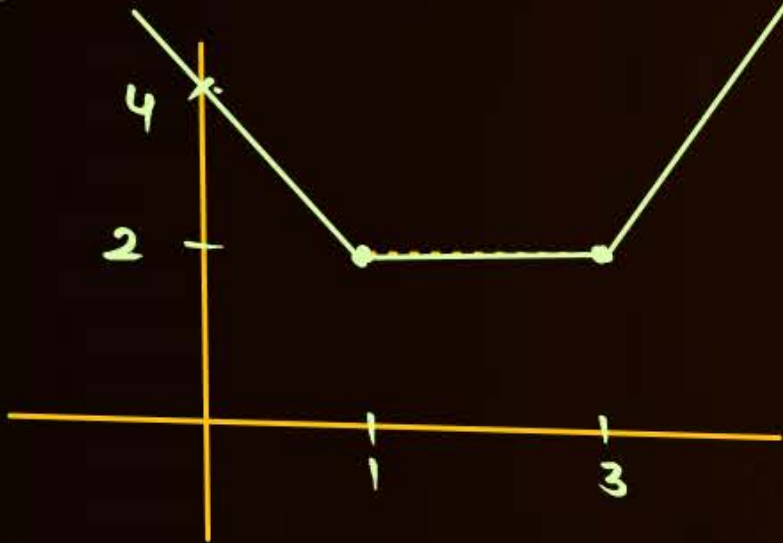
- (A) If $f(x) = k$ has no solution, then $k \in$
- (B) If $f(x) = k$ has one solution, then $k \in$
- (C) If $f(x) = k$ has two solution, then $k \in$
- (D) If $f(x) = k$ has more than two solution, then $k \in$

Column-II

- (p) $(2, 4)$
- (q) $(-\infty, -2) \cup (4, \infty)$
- (r) $(-2, 2) \cup \{4\}$
- (s) $\{-2, 2\}$

Ex: $y = |x-1| + |x-3|$ $\begin{array}{c} - & + & + \\ - & 1 & - & 3 & + \end{array}$

$$y = \begin{cases} -x+1-x+3 & x < 1 \\ x-1-x+3 & 1 \leq x \leq 3 \\ x-1+x-3 & x > 3 \end{cases} = \begin{cases} 4-2x & x < 1 \\ 2 & 1 \leq x \leq 3 \\ 2x-4 & x > 3 \end{cases}$$



$$f(x) = |x-1| + |x-3|$$

Range of $f(x) = [2, \infty)$

Yaxis pe Range milti hai

$$x=1 < x=2 < x=3$$

$$y = |x-1| + |x-2| + |x-3|$$

$y_{\min}$ @ $x=2$

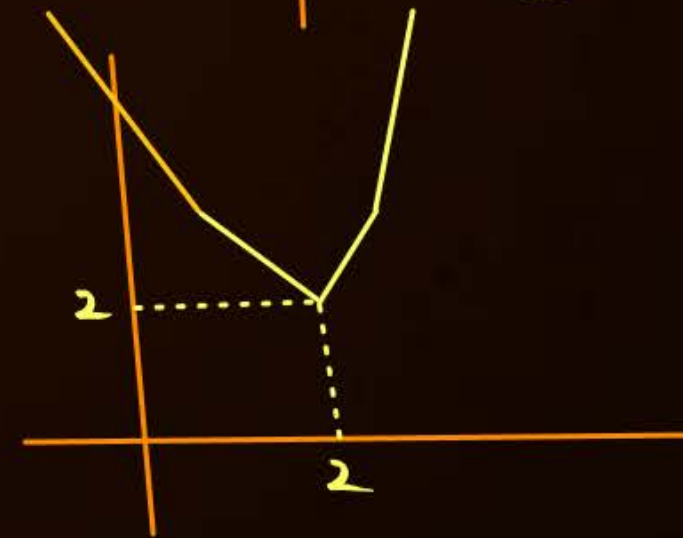
$$\text{Range } [1+0+1, \infty) = [2, \infty)$$

$$x=1 < \boxed{x=2 < x=3} < x=4$$

$$y = |x-1| + |x-2| + |x-3| + |x-4|$$

$y_{\min}$ @ $x \in [2, 3]$

$$\text{Range } [1+0+1+2, \infty) = [4, \infty)$$



QUESTION



$$y = |x - 1| + |x - 2| + |x - 3| \text{ and find } y_{\min}.$$

$$y_{\min} @ x=2$$

$$y_{\min} = 1 + 0 + 1 = 2$$

$$\text{Ex: } y = |x+1| + |x-3| + |x+5| + |x-4| + |x-7|$$
$$x = -5 < x = -1 < x = 3 < x = 4 < x = 7$$
$$y = |x+5| + |x+1| + |x-3| + |x-4| + |x-7|$$

$$y_{\min} \text{ at } x=3.$$

$$\text{Range } [8+4+0+1+4, \infty) = [17, \infty)$$

QUESTION



Given, $f(x) = |x| + 2|x - 1| + |x - 2| + |x - 4| + |x - 6| + 2|x - 10|$ where $x \in \mathbb{R}$. Find the minimum value of f .

$$y = |x| + |x-1| + |x-1| + |x-2| + |x-4| + |x-6| + |x-10| + |x-10|$$

$x=0 < x=1 \leq x=1 < x=2 < x=4 < x=6 < x=10 \leq x=10$

$$y_{\min} @ x \in [2, 4]$$

$$y_{\min} = 2 + 1 + 1 + 0 + 2 + 4 + 8 + 8 = 26$$

$$\text{Range } f(x) = [26, \infty)$$

Ex find range of $f(x) = |3x+4| + |2x-3|$

$$|-\frac{4}{3} - \frac{3}{2}| = |-\frac{8-9}{6}| = \frac{17}{6}$$

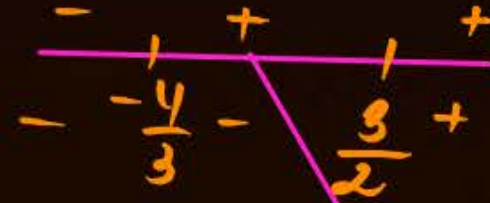
$$f(x) = 3|x + \frac{4}{3}| + 2|x - \frac{3}{2}|$$

$$f(x) = |x + \frac{4}{3}| + |x + \frac{4}{3}| + |x + \frac{4}{3}| + |x - \frac{3}{2}| + |x - \frac{3}{2}|$$

↑
 $y_{min} @ x = -\frac{4}{3}$

$$\begin{aligned} \text{Range} &= [0+0+0+\frac{17}{6}+\frac{17}{6}, \infty) \\ &= [\frac{17}{3}, \infty) \end{aligned}$$

$$y = |3x+4| + |2x-3|$$



$$y = \begin{cases} -3x-4-2x+3, & x \leq -\frac{4}{3} \\ 3x+4-2x+3, & -\frac{4}{3} < x < \frac{3}{2} \\ 3x+4+2x-3, & x > \frac{3}{2} \end{cases}$$

$$y = \begin{cases} -5x-1 & x \leq -\frac{4}{3} \\ x+7 & -\frac{4}{3} < x < \frac{3}{2} \\ 5x+1 & x > \frac{3}{2} \end{cases}$$

x	$-\frac{4}{3}$	$-\frac{2}{3}$
y	$\frac{17}{3}$	9

x	$-\frac{4}{3}$	0
y	$\frac{17}{3}$	7

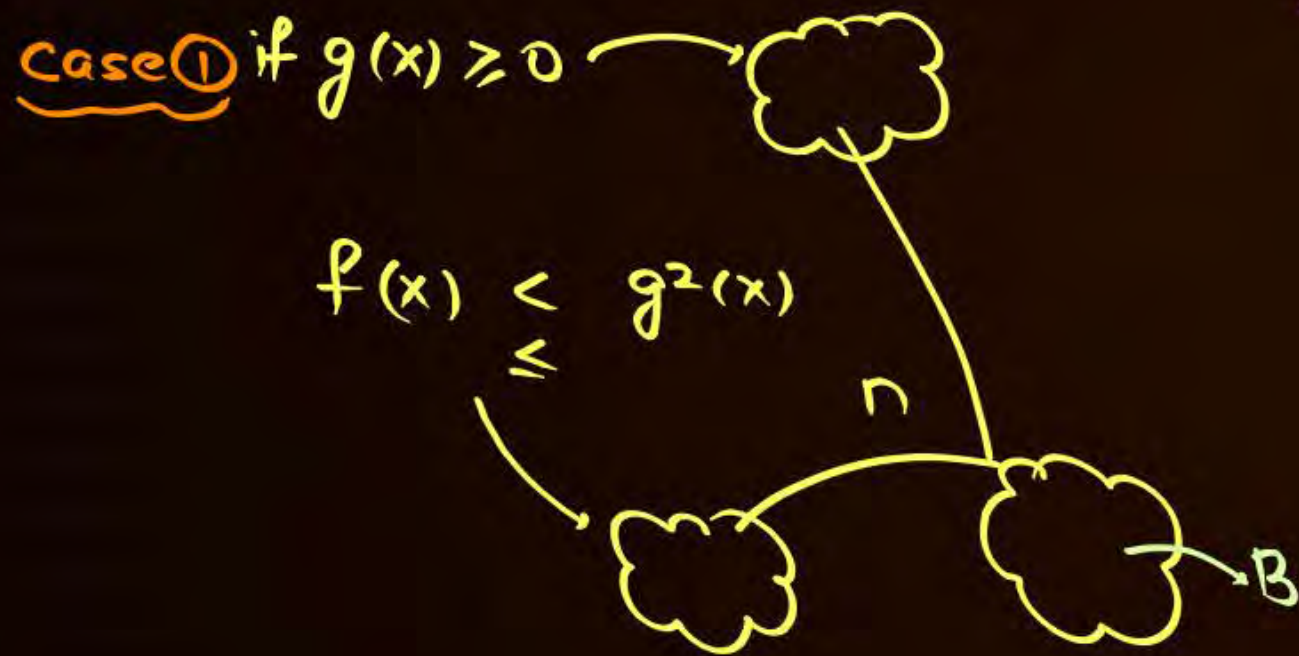




Irrational Inequalities

$$\sqrt{f(x)} < g(x) \\ \leq$$

$$f(x) \geq 0 \rightarrow \text{cloud} \text{---} \textcircled{A}$$



Final Ans: $A \cap B$

Case ②

$$\text{if } g(x) < 0 \rightarrow \text{cloud}$$
$$\sqrt{f(x)} < g(x)$$
$$\downarrow$$
$$x \in \phi$$
$$x \in \phi$$



Irrational Inequalities

$$\sqrt{f(x)} \geq g(x)$$

$$f(x) \geq 0 \rightarrow \text{cloud} \text{---} A$$

Case ① if $g(x) \geq 0$

$$f(x) \geq g^2(x)$$

Case ②

$$\text{if } g(x) < 0$$

$$\sqrt{f(x)} \geq g(x)$$

↓
always true

Final Ans: $A \cap B$



Irrational Inequalities

$$\sqrt{f(x)} \geq \sqrt{g(x)}$$
$$\leq$$

S.B.S

$$f(x) \geq g(x)$$
$$\leq$$

$$f(x) \geq 0 \quad \& \quad g(x) \geq 0$$



QUESTION



Solve for x if, $4 - x < \sqrt{x^2 - 2x}$

$$x^2 - 2x \geq 0$$

$$x(x-2) \geq 0$$

non-ve

$$x \in (-\infty, 0] \cup [2, \infty)$$

(A)

Ineq mai dono side hamesha odd power toh laga sakte hai but Even power ke liyaay dono sides negative nahi honi chahiyaay

case ① if $4 - x \geq 0 \Rightarrow x \leq 4$

$$4 - x < \sqrt{x^2 - 2x}$$

non-ve non-ve

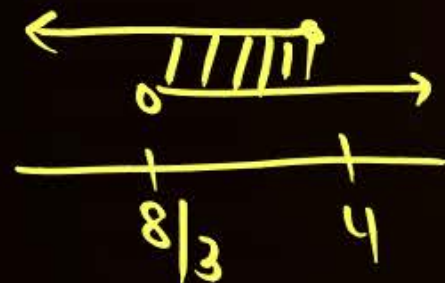
S.B.S

$$(4 - x)^2 < x^2 - 2x$$

$$16 + x^2 - 8x < x^2 - 2x$$

$$6x > 16$$

$$x > 8/3$$



$$x \in (8/3, 4]$$

case ② if $4 - x < 0 \Rightarrow x > 4$

$$4 - x < \sqrt{x^2 - 2x}$$

non-ve non-ve

always true.

$$x \in (4, \infty)$$

UNION

$$x \in (8/3, 4] \cup (4, \infty) = (8/3, \infty) \text{ --- (B)}$$

$A \cap B$

$$x \in (-\infty, 0] \cup [2, \infty)$$

$$x \in (8/3, \infty)$$



$$\text{Ans: } (8/3, \infty)$$

QUESTION



Solve for x, if $\sqrt{x^2 - 3x + 2} > x - 2$

$$x^2 - 3x + 2 \geq 0$$

$$(x-1)(x-2) \geq 0$$

$$x \in (-\infty, 1] \cup [2, \infty) \text{ --- (A)}$$

Case ① If $x-2 \geq 0 \Rightarrow x \geq 2$

non-ve $\sqrt{x^2 - 3x + 2} > x - 2$ non-ve

S.B.S $x^2 - 3x + 2 > x^2 - 4x + 4$

$$x > 2$$

$x \in (2, \infty)$

Case ② If $x-2 < 0 \Rightarrow x < 2$

$\sqrt{x^2 - 3x + 2} > x - 2$

non-ve $\downarrow$ always true

$x < 2$

UNION

$x \in (-\infty, \infty) - \{2\} \text{ --- (B)}$

Ans: $A \cap B = (-\infty, 1] \cup (2, \infty)$ Ans

QUESTION



Solution of the inequality, $x - 3 < \sqrt{x^2 + 4x - 5}$

Tah 5

- A** $(-\infty, -5] \cup [1, \infty)$
- B** $(-\infty, -5] \cup (7/5, \infty)$
- C** $(-5, 3]$
- D** $(7/5, \infty)$

QUESTION



Solve following inequalities

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$

Tah 06

SBS

(ii) $x - \sqrt{1 - |x|} < 0$

(iii) $\sqrt{x^2 - x - 6} < 2x - 3$

(iv) $\sqrt{x^2 - 6x + 8} \leq \sqrt{x + 1}$

(iii)

$x^2 - x - 6 \geq 0 \quad (x-3)(x+2) \geq 0$
 $x \in (-\infty, -2] \cup [3, \infty)$ — (A)

case ① if $2x - 3 \geq 0 \Rightarrow x \geq \frac{3}{2}$

$\sqrt{x^2 - x - 6} < 2x - 3$ — non-ve

$x^2 - x - 6 < 4x^2 + 9 - 12x$

$3x^2 - 11x + 15 > 0$

$\Delta < 0, a > 0$

always +ve i.e. $x \in \mathbb{R}$

Ans. $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$

Ans. $[-1, (\sqrt{5} - 1)/2]$

Ans. $x \in [3, \infty)$

Ans. $x \in \left[\frac{7 - \sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7 + \sqrt{21}}{2}\right]$

case ② if $2x - 3 < 0 \Rightarrow x < 3/2$

$\sqrt{x^2 - x - 6} < 2x - 3$ — -ve

Not possible

$x \in \emptyset$

$x \in [3/2, \infty)$

$x \in [3/2, \infty)$

(B)

Ans: $A \cap B = [3, \infty)$



$$(iv) \sqrt{x^2 - 6x + 8} \leq \sqrt{x+1}$$

SBS

$$x^2 - 6x + 8 \leq x + 1$$

$$x^2 - 7x + 7 \leq 0$$

$$\left(x - \frac{7 + \sqrt{21}}{2}\right) \left(x - \frac{7 - \sqrt{21}}{2}\right) \leq 0$$

$$x \in \left[\frac{7 - \sqrt{21}}{2}, \frac{7 + \sqrt{21}}{2}\right]$$

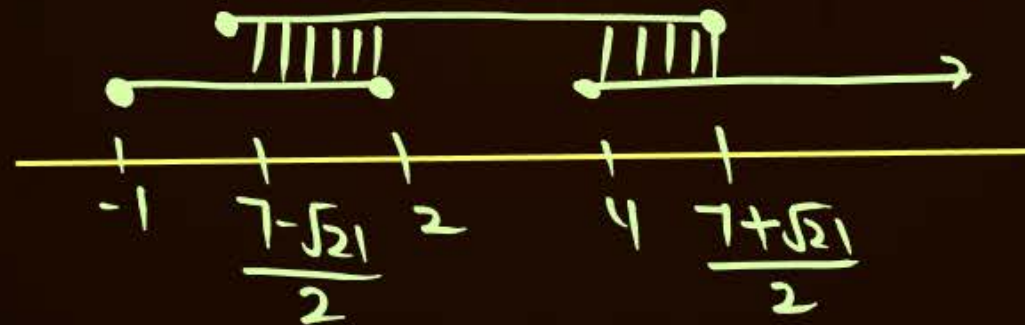
$$x = \frac{7 \pm \sqrt{21}}{2}$$

$$x^2 - 6x + 8 \geq 0 \quad \& \quad x + 1 \geq 0$$

$$(x-2)(x-4) \geq 0 \quad \& \quad x \geq -1$$

$$x \in (-\infty, 2] \cup [4, \infty) \quad \cap$$

$$x \in [-1, 2] \cup [4, \infty)$$



$$x \in \left[\frac{7 - \sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7 + \sqrt{21}}{2}\right]$$

Let $S = \{x: x \in \mathbb{R} \text{ and } (\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10\}$. Then $n(S)$ is equal to

A 6

B 4

C 0

D 2

$$\log_{10} \left((\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} \right) = \log_{10} 10$$

~~$$\log_{10} (\sqrt{3} + \sqrt{2})^{x^2-4} + \log_{10} (\sqrt{3} - \sqrt{2})^{x^2-4} = \log_{10} 10$$~~

$$\log_a(b+c) \neq \log_a b + \log_a c$$

$$\text{Ex: } \log_2(8+4) \neq \log_2 8 + \log_2 4 = 3+2=5$$

$$\quad \quad \quad \parallel$$

$$\quad \quad \quad \log_{12} 2$$

$$\log_{10} (\sqrt{3} + \sqrt{2})^{x^2-4} + \log_{10} (\sqrt{3} - \sqrt{2})^{x^2-4} = \log_{10} 10$$

Gadho/Gadhiyon aisa
naa kro

Let $S = \{x: x \in \mathbb{R} \text{ and } (\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10\}$. Then $n(S)$ is equal to

A 6

B 4

C 0

D 2

$$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}) = 3-2=1$$

$$\sqrt{3}-\sqrt{2} = \frac{1}{\sqrt{3}+\sqrt{2}}$$

$$(\sqrt{3}+\sqrt{2})^{x^2-4} + \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^{x^2-4} = 10$$

$$\text{Let } (\sqrt{3}+\sqrt{2})^{x^2-4} = t \quad (\sqrt{3}+\sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$$

$$t + \frac{1}{t} = 10$$

$$t^2 - 10t + 1 = 0$$

$$t = \frac{10 \pm \sqrt{96}}{2} = \frac{10 \pm 4\sqrt{6}}{2}$$

$$t = 5 \pm 2\sqrt{6} = (\sqrt{3} \pm \sqrt{2})^2$$

$$\log_{10} (\sqrt{3}+\sqrt{2})^{x^2-4} + \log_{10} (\sqrt{3}-\sqrt{2})^{x^2-4} = \log_{10} 100$$

Gadho/Gadhiyon aisa
naa kro

$$(\sqrt{3} + \sqrt{2})^{x^2-4} = (\sqrt{3} + \sqrt{2})^2 \text{ or } (\sqrt{3} - \sqrt{2})^2$$

$$x^2 - 4 = 2 \quad \text{or} \quad (\sqrt{3} + \sqrt{2})^{x^2-4} = (\sqrt{3} - \sqrt{2})^2 = \frac{1}{(\sqrt{3} + \sqrt{2})^2} = (\sqrt{3} + \sqrt{2})^{-2}$$

$$x = \pm \sqrt{6}$$

$$x^2 - 4 = -2$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

$$\text{Ans: } S = \{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$$

$$n(S) = 4$$

Let $S = \{x \in \mathbb{R} : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$. Then the number of elements in S is :

- A** 4
- B** 0
- C** 2
- D** 1

The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is

- A** 0
- B** 1
- C** 3
- D** 2

QUESTION

Tan 09



Solve: $\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$

The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$ ($x > 0$) is equal to:

- A** 9
- B** 12
- C** 4
- D** 10



Sabse Important Baat



Sabhi Class Illustrations Retry Karnay hai...



Home Challenge-08



Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^\lambda$, then find the value of $(2\lambda - 15)$. [Ans. 6]

Today's BPP

Bumper Practice Problems



$$1. \sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$$

$$2. \sqrt{3x-2} < \sqrt{x+4}$$

$$3. \sqrt{4x-3} < \sqrt{2x+5}$$

$$4. \sqrt[3]{(3x-5)} < (x-1)$$

$$5. \sqrt[3]{3x-2} < x$$

$$6. \sqrt{(x+14)} < (x+2)$$

$$7. \sqrt{2x-2} < x-1$$

$$8. \sqrt{-x^2+4x-3} > (6-2x)$$

$$9. \sqrt{x-2} + \sqrt{x-1} > 2$$

$$10. 7\sqrt{x} + 8\sqrt{-x} + \frac{15}{x^3} = 98$$

$$11. \sqrt{2x-4} - \sqrt{x+5} = 1$$

$$12. \sqrt{x-1} + \sqrt{2x+6} = 6$$

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

5. $x \in (-2, 1) \cup (1, \infty)$

6. $x \in (2, \infty)$

7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

9. $x \in \left(\frac{41}{16}, \infty\right)$



Homework From Module

Prarambh (Topicwise) : Q1 to Q25

Prabal (JEE Main Level) : Q1 to Q33

Solution to Previous TAH

QUESTION



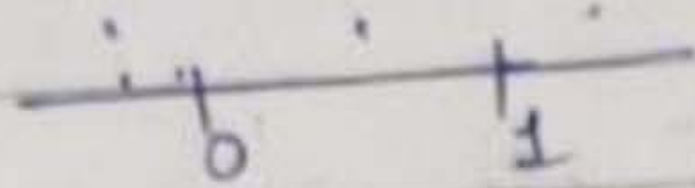
Solve: $|x| + |x - 1| \geq 7$

Lecture 15 : 08/05/2025

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Date:

IAH-01:



(i) If $x \leq 0$,
 $-x - x + 1 \geq 7$

$$-2x \geq 6$$

$$x \leq -3$$

$$\Rightarrow \boxed{x \in (-\infty, -3)}$$

(ii) If $0 < x < 1$,
 $+x - x + 1 \geq 7$
 $1 \geq 7$

(Not possible)

$$\Rightarrow x \in \phi$$

(iii) If $x \geq 1$,
 $x + x - 1 \geq 7$

$$2x \geq 8$$

$$x \geq 4$$

$$\Rightarrow \boxed{x \in (4, \infty)}$$

$$\boxed{x \in (-\infty, -3) \cup (4, \infty)} \text{ Ans.}$$

Tah-0

81

$$|x| + |x-1| \geq 7$$

$$x < 0$$

$$-x - x + 1 \geq 7$$

$$-2x \geq 6$$

$$x \leq -3$$

$$x \in (-\infty, -3]$$

$$0 < x < 1$$

$$+x - x + 1 \geq 7$$

$$1 \geq 7 \quad \text{X (NP)}$$

$$x > 1$$

$$x + x - 1 \geq 7$$

$$2x \geq 8$$

$$x \geq 4$$

$$x \in [4, \infty)$$

$$x \in (-\infty, -3] \cup [4, \infty)$$

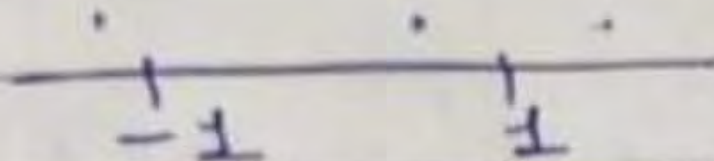
QUESTION



Solve: $|x + 1| + |x - 1| > 2$

#

IAH-02:



(i) If $x \leq -1$,
 $\rightarrow x+1-x+1 > 2$
 $-2x > 2$
 $x < -1$
 $\Rightarrow x \in (-\infty, -1)$

(ii) If $-1 < x < 1$
 $x+1-x+1 > 2$
 $2 > 2$
 (Not possible)
 $\Rightarrow x \in \emptyset$

(iii) If $x \geq 1$
 $x+1-x+1 > 2$
 $2x > 2$
 $x > 1$
 $\Rightarrow x \in (1, \infty)$

$\{x \in (-\infty, -1) \cup (1, \infty)\}$ Ans

Method-2: Using triangle inequality.

$$|x+1| + |x-1| > 2$$

$$|a| + |b| > |a-b| \iff ab > 0$$

$$\Rightarrow (x+1) \cdot (x-1) > 0$$

$\Rightarrow \{x \in (-\infty, -1) \cup (1, \infty)\}$ Ans

Tah.02

$$\begin{array}{cc} T_1 & T_2 \\ |x+1| + |x-1| > 2 \end{array}$$

$$\begin{array}{ccc} T_1 & -ve & +ve & +ve \\ T_2 & -ve & -ve & +ve \end{array}$$

Case I $\rightarrow x \leq -1$ Case II $\rightarrow -1 < x < 1$ Case III $\rightarrow x \geq 1$

$$-(x+1) - (x-1) > 2 \quad x+1 - (x-1) > 2 \quad (x+1) + (x-1) > 2$$

$$-x-1-x+1 > 2 \quad x+1-x+1 > 2 \quad x+1+x-1 > 2$$

$$-2x > 2$$

$$2 > 2$$

$$2x > 2$$

$$-2x-2 > 0$$

No sol.

$$x > 1$$

$$-2(x+1) > 0$$

$$x \in (1, \infty)$$

$$x+1 < 0$$

$$x \in (-\infty, -1)$$

$$x \in (-\infty, -1) \cup (1, \infty)$$

QUESTION



Solve: $|x - 6| > |x^2 - 5x + 9|$

Tqn 03
8

$$|x-6| > |x^2-5x+9|$$

$$(x-6)^2 > (x^2-5x+9)^2$$

$$(x-6+x^2-5x+9)(x-6-x^2+5x-9) > 0$$

$$(x^2-4x+3)(-x^2+6x-15) > 0$$

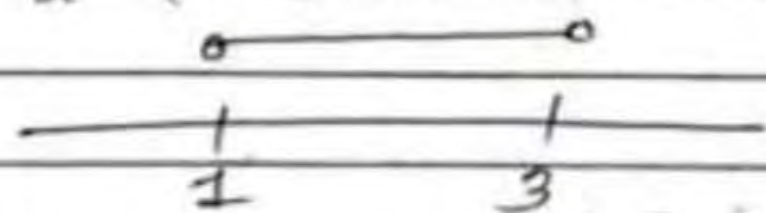
$$(x^2-4x+3)(x^2-6x+15) < 0$$

$\rightarrow D < 0$

$$(x^2-3x-x+3) \cdot x^2 < 0$$

$$x(x-3) - 1(x-3) < 0$$

$$x(x-1)(x-3) < 0$$



$$\boxed{x \in (-1, +3)}$$

TAM-03:

$$(x-6)^2 > (x^2-5x+9)^2$$

$$(x-6+x^2-5x+9)(x-6-x^2+5x-9) > 0$$

$$(x^2-4x+3) \cdot (-x^2+6x-15) > 0$$

$$(x-3)(x-1)(x^2-6x+15) < 0 \Rightarrow (x-3) \cdot (x-1) < 0$$

$\hookrightarrow D < 0, a > 0$
(always +ve)

$$\{x \in (1, 3)\} \text{ Ans}$$

QUESTION

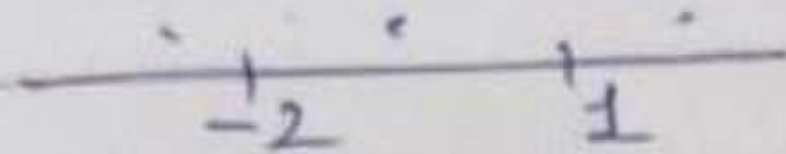


Solve: $(|x - 1| - 3)(|x + 2| - 5) < 0$

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TAH-04:



(i) If $x \leq -2$,

(ii) If $-2 < x < 1$

(iii) If $x \geq 1$

$$\begin{array}{lll}
 (-x+1-3) \cdot (-x-2-5) < 0 & (-x+1-3)(x+2-5) < 0 & (x-1-3) \cdot (x+2-5) < 0 \\
 (x+2) \cdot (x+7) < 0 & (x+2) \cdot (x-3) > 0 & (x-4)(x-3) < 0 \\
 x \in (-7, -2) & x \in (-\infty, -2) \cup (3, \infty) & x \in (3, 4) \\
 \Rightarrow \boxed{x \in (-7, -2)} & \Rightarrow x \in \emptyset & \Rightarrow \boxed{x \in (3, 4)}
 \end{array}$$

$$\boxed{x \in (-7, -2) \cup (3, 4)} \quad \underline{\underline{Ans}}$$

TAH-04

$$(1x-11-3)(1x+21-5) < 0.$$

Either $1x-11-3 > 0$ & $1x+21-5$ X This method was already told by sir...

(H-7)

Case ① when $x \leq -2$.

$$(-x+1-3)(-x-2-5) < 0.$$

$$(x+2)(x+7) < 0.$$

$$(-7, -2)$$

When $-2 < x \leq 1$

$$(-x+1-3)(x+3) < 0.$$

$$(x+2)(x-3) > 0.$$

$$(-\infty, -2) \cup (3, \infty). \text{ Case Rejected.}$$

When $x \geq 1$.

$$(x-4)(x-3) < 0.$$

$$x \in (3, 4)$$

$$(-7, -2) \cup (3, 4) \text{ Ans}$$

(a) $|x^2 - 9| + |16 - x^2| = 7$

Ans: $x \in [-4, -3] \cup [3, 4]$

(b) Solve: $|(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$

Ans: $x < -7/3$

TAH-05:



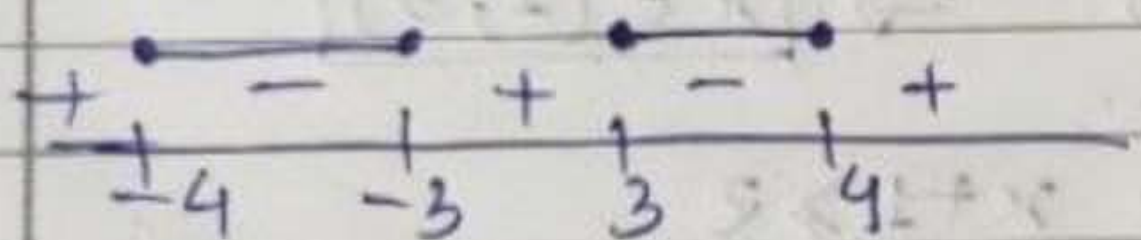
(a) $|x^2-9| + |16-x^2| = 7$

$|a| + |b| = |a+b| \iff ab \geq 0$

$(x^2-9) \cdot (16-x^2) \geq 0$

$(x^2-9)(x^2-16) \leq 0$

$(x-3)(x+3)(x-4)(x+4) \leq 0$



$\Rightarrow x \in [-4, -3] \cup [3, 4]$

Ans.

(b) $|(x^2+2x+2)+(3x+7)| < |x^2+2x+2| + |3x+7|$

$|a+b| < |a| + |b| \iff ab < 0$

$(x^2+2x+2) \cdot (3x+7) < 0 \Rightarrow 3x+7 < 0$

$(D < 0, a > 0)$

$x < -\frac{7}{3}$

Ans.

Ex 105(0) $|x^2 - 9| + |16 - x^2| = 7$

$\downarrow$ $\downarrow$
 a b

$$|a| + |b| \geq |a + b|$$

$$|x^2 - 9| + |16 - x^2| \geq |x^2 - 9 + 16 - x^2| = 7$$

holds equality when -

$$ab \geq 0$$

$$(x^2 - 9)(16 - x^2) \geq 0$$

$$(x - 3)(x + 3)(x^2 - 16) \leq 0$$

$$(x - 3)(x + 3)(x - 4)(x + 4) \leq 0$$

$$\begin{array}{ccccccc} + & - & + & - & + \\ -4 & -3 & 3 & 4 \end{array}$$

$$x \in [-4, -3] \cup [3, 4]$$

(b) $|(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$

$\downarrow$ $\downarrow$
 a b

$$|a + b| < |a| + |b|$$

strictly increase when $ab < 0$

Therefore -

$$(x^2 + 2x + 2)(3x + 7) < 0$$

$$\begin{array}{l} \hookrightarrow 0 < 0 \\ a > 1 \text{ (always +ve)} \end{array}$$

$$(3x + 7) < 0$$

$$x < -\frac{7}{3}$$

$$x \in \left(-\infty, -\frac{7}{3}\right)$$

Solution to Previous KTKs

Column-I

- (A) $\frac{\log_2 32}{\log_3 \sqrt{243}}$
- (B) $\frac{2\log 6}{\log 12 + \log 3}$
- (C) $\log_{1/4} \left(\frac{1}{16}\right)^{-2}$
- (D) $\frac{\log_5 16 - \log_5 4}{\log_5 128}$

Column-II

- (P) positive integer
- (Q) negative integer
- (R) rational but not integer
- (S) Prime

Ans. A $\rightarrow$ P, S; B $\rightarrow$ P; C $\rightarrow$ Q; D $\rightarrow$ R

$$A.) \log_{10} x \cdot \log_{10} x = \log_{10} 100x$$

$$\log_{10}^2 x = 2 + \log_{10} x$$

$$t^2 - t - 2 = 0$$

$$t^2 - 2t + t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$\log_{10} x = 2 \text{ or } \log_{10} x = -1$$

$$x = 100 \text{ or } \frac{1}{10}$$

$$x_1 x_2 = 10$$

Q.15

$$B.) 9 - 2^x = 2^{3-x}$$

$$9 - 2^x = \frac{2^3}{2^x}$$

$$9 - t = \frac{8}{t}$$

$$9t - t^2 = 8$$

$$0 = t^2 - 9t + 8$$

$$0 = t^2 - 8t - t + 8$$

$$0 = (t-8)(t-1)$$

$$2^x = 2^3 \text{ or } 2^x = 2^0 \quad \text{Q}$$

$$x = 3 \text{ or } 0 \quad x_1, x_2 = 0$$

$$c.) \log_{\frac{1}{8}} \left(\log_{\frac{1}{4}} \left(\log_{\frac{1}{2}} x \right) \right) = \frac{1}{3}$$

$$\log_{\frac{1}{4}} \left(\log_{\frac{1}{2}} x \right) = \frac{1}{2}$$

$$\log_{\frac{1}{2}} x = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{2}} \quad (P)$$

$$p.) \log_b a = 3, \quad \log_b c = -4, \quad a^{3x} = c^{x-1}$$

$$a = b^3, \quad c = b^{-4}$$

$$b^{9x} = b^{4-4x}$$

$$b^{9x} = \frac{b^4}{b^{4x}}$$

$$b^{13x} = b^4$$

$$13x = 4$$

$$x = \frac{4}{13}$$

$$p+q = 17 \quad (Q, R)$$

$$\textcircled{A} \quad \frac{\log_2 2^5}{\log_3 3^{5/2}} \Rightarrow \frac{5 \times 2}{5} \Rightarrow \textcircled{2} \Rightarrow \boxed{P, S}$$

$$\textcircled{B} \quad \frac{2 \log 6}{\log 12 + \log 3} \Rightarrow \frac{\log 6^2}{\log 6^2} \Rightarrow \textcircled{1} \Rightarrow \boxed{P}$$

$$\textcircled{C} \quad \log_{\frac{1}{4}} (16)^2 \Rightarrow \log_{4^{-1}} (4)^4 \Rightarrow \textcircled{-4} \Rightarrow \boxed{Q}$$

$$\textcircled{D} \quad \frac{\log_5 4}{\log_5 2^7} \Rightarrow \frac{2 \log_5 2}{7 \log_5 2} \Rightarrow \textcircled{\frac{2}{7}} \Rightarrow \boxed{R}$$

Solⁿ

$$(A) \frac{\log_2 32}{\log_3 \sqrt{243}} = \frac{\log_2 (2)^5}{\log_3 \sqrt{3^6}} = \frac{5}{\frac{5}{2}} = \boxed{2}$$

3⁵

(P) & (S)

$$(B) \frac{2 \log 6}{\log 12 + \log 3} = \frac{2 \log 6}{\log 36} = \frac{2 \log 6}{\log (6)^2} = \frac{2 \log 6}{2 \log 6} = \boxed{1}$$

(P)

$$(C) \log_{1/9} \left(\frac{1}{16} \right)^{-2} = \log_{1/9} \left(\left(\frac{1}{9} \right)^2 \right)^{-2} = \log_{1/9} \left(\frac{1}{9} \right)^{-4} = \boxed{-9}$$

(Q)

$$(D) \frac{\log_5 16 - \log_5 9}{\log_5 128} = \frac{\log_5 4}{\log_5 (2)^7} = \frac{\log_5 (2)^2}{\log_5 (2)^7} = \frac{2 \log_5 2}{7 \log_5 2} = \boxed{\frac{2}{7}}$$

(R)

QUESTION**(KTK 2)**

$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

Ans. $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

$$\Rightarrow \frac{1}{x-1} - \frac{1}{x-4} + \frac{4}{x-3} - \frac{4}{x-2} < \frac{1}{30}$$

$$\frac{(x-4) - (x-1)}{(x-1)(x-4)} + 4 \left[\frac{(x-2) - (x-3)}{(x-3)(x-2)} \right] < \frac{1}{30}$$

$$\frac{-3}{(x-1)(x-4)} + \frac{4}{(x-3)(x-2)} < \frac{1}{30}$$

$$\frac{-3}{x^2 - 5x + 4} + \frac{4}{x^2 - 5x + 6} < \frac{1}{30}$$

$$x^2 - 5x = t$$

$$\frac{-3}{t+4} + \frac{4}{t+6} - \frac{1}{30} < 0$$

$$\frac{-3(30)(t+6) + 4(30)(t+4) - (t+4)(t+6)}{(t+4)(t+6)(30)} < 0$$

$$\frac{-90t - 540 + 120t + 480 - t^2 - 10t - 24}{(t+4)(t+6)} < 0$$

$$\text{Ans. } x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$$

$$\frac{-t^2 + 20t - 84}{(t+4)(t+6)} < 0$$

$$\Rightarrow \frac{t^2 - 20t + 84}{(t+4)(t+6)} > 0$$

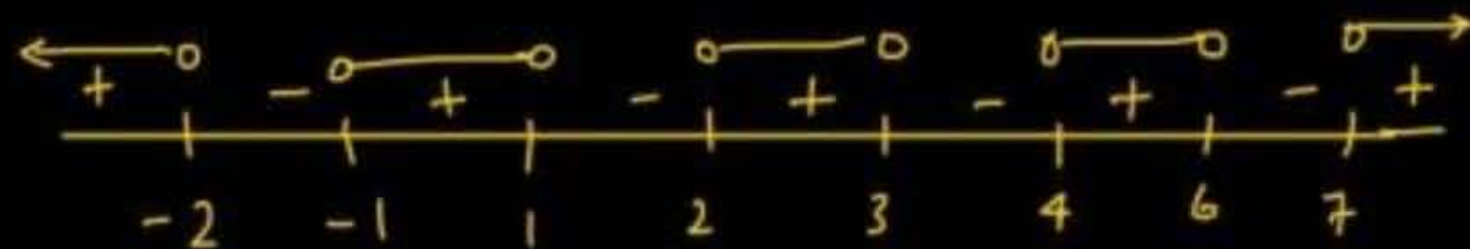
$$\Rightarrow \frac{t^2 - 14t - 6t + 84}{(t+4)(t+6)} > 0$$

$$\Rightarrow \frac{(t-14)(t-6)}{(t+4)(t+6)} > 0$$

$$\frac{(x^2 - 5x - 14)(x^2 - 5x - 6)}{(x^2 - 5x + 4)(x^2 - 5x + 6)} > 0$$

$$\frac{(x^2 - 7x + 2x - 14)(x^2 - 6x + x - 6)}{(x^2 - 5x + 4)(x^2 - 5x + 6)} > 0$$

$$\frac{(x-7)(x+2)(x-6)(x+1)}{(x-1)(x-4)(x-3)(x-2)} > 0$$



$$x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$$

The equation $\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has

- A** no integral solution
- B** one natural solution
- C** two real solutions
- D** one irrational solution

KTK-03

The eqⁿ $\frac{\log_8 (8/x^2)}{(\log_8 x)^2} = 3$

 $\Rightarrow$

$$\frac{1 - \log_8 x^2}{(\log_8 x)^2} = 3$$

$$\Rightarrow 1 - 2 \log_8 x = 3(\log_8 x)^2$$

$$\text{let } \log_8 x = t$$

$$1 - 2t = 3t^2$$

$$3t^2 + 2t - 1 = 0$$

$$3t^2 + 3t - t - 1 = 0$$

$$3t(t+1) - 1(t+1) = 0$$

$$(3t-1)(t+1) = 0$$

$$t = \frac{1}{3}, -1$$

$$\log_8 x = \frac{1}{3}$$

$$x = 2$$

$$\log_8 x = -1$$

$$x = \frac{1}{8}$$

B) one natural solⁿ

C) 2 real solⁿ

$$\frac{\log_8 \left(\frac{8}{x^2} \right)}{(\log_8 x)^2} = 3 \text{ has}$$

$$\log_8 \left(\frac{8}{x^2} \right) = 3(\log_8 x)^2$$

$$(\log_8 8 - \log_8 x^2) = 3(\log_8 x)^2$$

$$\text{Let } \log_8 x = t$$

$$(1 - 2t) = 3[t]^2$$

$$3t^2 + 2t - 1 = 0$$

$$3t^2 + 3t - t - 1 = 0$$

$$3t[t+1] - 1[t+1] = 0$$

$$(t+1)(3t-1) = 0$$

$$\boxed{t = -1}$$

$$\boxed{t = \frac{1}{3}}$$

$$\log_8 x = -1$$

$$\boxed{x = \frac{1}{8}}$$

$$\log_8 x = \frac{1}{3}$$

$$\boxed{x = 2}$$



For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

- A** no real solution
- B** one prime solution
- C** one integral solutions
- D** no irrational solution

Ans. A, B, D

$$\frac{\log_3 x}{\log_3 3\sqrt{x}} + \frac{\log_3 \sqrt{x}}{\log_3 3x} = 0$$

$$\frac{\log_3 x}{1 + \log_3 \sqrt{x}} + \frac{\log_3 \sqrt{x}}{1 + \log_3 x} = 0$$

$$\frac{\log_3 x}{1 + \frac{1}{2} \log_3 x} + \frac{\frac{1}{2} \log_3 x}{1 + \log_3 x} = 0$$

$$\frac{2x}{2+x} + \frac{x}{2(1+x)} = 0$$

$$\frac{2x \cdot (2)(1+x) + x(2+x)}{2(1+x)(2+x)} = 0$$

$$\frac{4x + 4x^2 + 2x + x^2}{2(1+x)(2+x)} = 0$$

$$5x^2 + 6x = 0$$

$$x(5x+6) = 0$$

$$x = 0, -\frac{6}{5}$$

$$\log_3 x = 0, -\frac{6}{5} \Rightarrow x = 1, (3)^{-6/5}$$

Ans: (a), (b), (d)

$$\boxed{x = 1, \frac{1}{3^{6/5}}}$$



If $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 (x^2) + \log_8 (x^3) + \log_{16} (x^4) = 40$ then x is equal to

KTK-05 If $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$, then x is equal to

$$\Rightarrow \log_2 x + \log_2 x + \log_2 x + \log_2 x + \log_2 x = 40$$

$$5 \log_2 x = 40$$

$$x = 2^8$$

$$= 256$$

KTK-05

DATE _____
PAGE _____

$$\log_{\sqrt{x}} x$$

$$\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 x^2 + \log_8 x^3 + \log_{16} x^4 = 40$$

$$2 \log_2 \sqrt{x} + \log_2 x + \frac{1}{2} \log_2 x^2 + \frac{1}{3} \log_2 x^3 + \frac{1}{4} \log_2 x^4 = 40$$

$$\log_2 x + \log_2 x + \log_2 x + \log_2 x + \log_2 x = 40$$

$$5t = 40$$

$$t = \frac{40}{5} = 8$$

$$\log_2 x = 8$$

$$\boxed{x = 2^8 = 256}$$

Column-I		Column-II	
(A)	If x_1 and x_2 satisfy the equation $x^{\log_{10} x} = 100x$ then the value of $x_1 x_2$ equals	(P)	irrational
(B)	Sum of the squares of the roots of the equation $\log_2(9 - 2^x) = 3 - x$ is	(Q)	rational
(C)	If $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$ then x is	(R)	prime
(D)	Let $\log_b a = 3, \log_b c = -4$. If the value of x satisfying the equation $a^{3x} = c^{x-1}$ is expressed in the form p/q , where p and q are relatively prime then $p + q$ is	(S)	composite

Ans. A $\rightarrow$ Q, S; B $\rightarrow$ Q, S; C $\rightarrow$ P; D $\rightarrow$ QR

KTk-06

(a) $x^{\log_{10} x} = 100x$

Taking log both sides

$$\log_{10} x \log_{10} x = \log_{10} 100x$$

$$(\log_{10} x)^2 = \log_{10} 100 + \log_{10} x$$

$$(\log_{10} x)^2 = 2 + \log_{10} x$$

Let $(\log_{10} x) = t$

$$t^2 = 2 + t$$

$$\log_{10} x = 2$$

$$x_1 = 100$$

$$\log_{10} x = -1$$

$$x_2 = \frac{1}{10}$$

$$t^2 - t - 2 = 0$$

$$t^2 - 2t + t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$t = +2, -1$$

$$x_1 \cdot x_2 = 100 \times \frac{1}{10} = 10 //$$

$$A \rightarrow 5, 8$$

(b) $\log_2 (9 - 2^x) = 3 - x$

$$2^{(3-x)} = 9 - 2^x$$

$$2^3 = 9 - 2^x$$

$$8 = 9 - 2^x \Rightarrow 2^x = 1$$

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$$2^x = t$$

$$t^2 - 9t + 8 = 0 \quad \cancel{9t^2 - 8t - t^2}$$

$$t^2 - 8t - t + 8 = 0 \quad \cancel{8t^2 + 9t - 0}$$

$$(t-8)(t-1) = 0 \quad \cancel{t^2 - 9t}$$

$$\boxed{t-8=1}$$

$$2^x = 8$$

$$\boxed{x=3}$$

$$2^x = 1$$

$$\boxed{x=0}$$

$$\boxed{B \rightarrow Q, S}$$

$$3^2 + 0^2 = 9$$

$$\textcircled{c} \quad \log_{\frac{1}{8}} \left(\log_{\frac{1}{4}} \left(\log_{\frac{1}{2}} x \right) \right) = \frac{1}{3}$$

$$\log_{\frac{1}{4}} \left(\log_{\frac{1}{2}} x \right) = \left(\frac{1}{8} \right)^{\frac{1}{3}}$$

$$\log_{\frac{1}{2}} x = \frac{1}{4} \left(\frac{1}{2} \right)$$

$$\log_{\frac{1}{2}} x = \frac{1}{2}$$

$$x = \left(\frac{1}{2} \right)^{\frac{1}{2}}$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

$$\boxed{C \rightarrow P}$$

$$\log_b a = 3 \quad \log_b c = -4 \quad , \quad a^{3x} = c^{x-1}$$

$$a^{3x} = c^{x-1}$$

Taking log both sides

$$3x \log_b a = (x-1) \log_b c$$

$$3x(3) = (x-1)(-4)$$

$$9x = -4x + 4$$

$$13x = 4$$

$$4+13$$

$$\boxed{17}$$

$$\boxed{D \rightarrow R}$$

$$\boxed{x = \frac{4}{13}}$$

РТХ06
8

$$A \quad 2^{\log_{10} x} = 100x$$

$$(\log_{10} x)^2 = \log_{10} 10^2 + \log_{10} x$$

$$t^2 = 2 + t$$

$$t^2 - t - 2 = 0$$

$$t = -1$$

$$x = \frac{1}{10}$$

$$t = 2$$

$$x_2 = 100$$

$$x_1 x_2 = 10$$

$$(B) \quad \log_2 (9 - 2^x) = 3 - x$$

$$2 \quad 9 - 2^x = \frac{8}{2^x}$$

$$9 - t = \frac{8}{t}$$

$$9t - t^2 = 8$$

$$t^2 - 9t + 8 = 0$$

$$t^2 - 8t - t + 8 = 0$$

$$t(t-8) - 1(t-8) = 0$$

$$t = 1$$

$$t = 8$$

$$x = 0$$

$$x = 3$$

(C)

$$\log_{\frac{1}{4}}(\log_{\frac{1}{2}} x) = 2^{-3 \times \frac{1}{2}}$$

$$\log_{\frac{1}{2}} x = 2^{-2 \times \frac{1}{2}}$$

$$x = 2^{-1 \times \frac{1}{2}}$$

$$\log_{\frac{1}{2}} x = \frac{1}{2}$$

$$x = \left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

(D)

$$\log_b a = 3 \quad , \quad \log_b c = -4$$

$$\Rightarrow \boxed{a = b^3}$$

$$b \quad \boxed{c = \frac{1}{b^4}}$$

$$a^{3x} = c^{x-1}$$

a

$$\boxed{a^3}^x$$

$$(a)^{3x} = \frac{(c)^{x-1}}{c}$$

$$(b^3)^{3x} = \frac{(b^{-4})^{x-1}}{b^{-4}}$$

$$b^{9x} = \frac{b^4}{b^{4x}}$$

$$b^{9x} = b^{4-4x}$$

$$9x = 4 - 4x$$

$$13x = 4$$

$$\boxed{x = \frac{4}{13}}$$

$$\boxed{p = 4}$$

$$\boxed{q = 13}$$



Home Challenge-07



Find the sum of all the integral solution(s) of the equation $3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}} \right)^2$. [Ans. 3]

Home Challenge-07

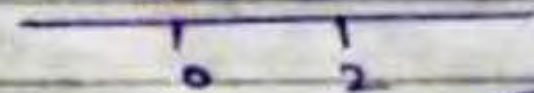
Find the sum of all integral solⁿ(s) of the eqⁿ

$$3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}} \right)^2$$

$$\Rightarrow 3^{|x|} = 3^{2-|x-2|}$$

bases same, powers can be compared

$$|x| = 2 - |x-2| \Rightarrow |x| + |x-2| - 2 = 0$$



(I)

$$x \leq 0$$

$$-x - x + 2 - 2 = 0$$

$$-2x = 0$$

$$x = 0$$

(II)

$$0 < x < 2$$

$$x - x + x - 2 = 0$$

$$\forall x \in \mathbb{R}$$

$$x \in (0, 2)$$

(IV)

$$x \geq 2$$

$$x + x - 2 - 2 = 0$$

$$2x = 4$$

$$x = 2$$

$$x \in [0, 2]$$

∴ integral values of $x = 0, 1, 2$

$$\text{Sum} = 0 + 1 + 2 = 3$$

Home challenge-07
8

$$3^{|x|} = \left(\frac{3}{\sqrt{3}} \right)^{|x-2|}$$

$$3^{|x|} = 3^{2-|x-2|}$$

$$|x| = 2 - |x-2|$$

$$x \leq 0$$

$$-x = 2 + x - 2$$

$$2x = 0$$

$$x = 0 \checkmark$$

$$0 < x \leq 2$$

$$x = 2 + x - 2$$

$$x = x$$

$$x \in (0, 2] \checkmark$$

$$x > 2$$

$$x = 2 - x - 2$$

$$2x = 0$$

$$x = 0 \times$$

integral solⁿ $x = \{1, 0, 2\}$

sum = 3 A

Solution to Previous BPPs

Bumper Practice Problems



Solve the following inequality for x .

(a) $|x| > 2$

(b) $|x - 1| > 3$

(c) $|x - 2| < 1$

(d) $|x + 1| \geq 2$

(e) $|x - 1| \leq 5$

(f) $|2x - 3| > 7$

(g) $|3x + 5| < 2$

(h) $|4x + 6| > 5$

(i) $|2x - 3| > -2$

(j) $|4x - 9| \leq 7$

(k) $|3x + 5| \geq 2$

(l) $|2x + 3| \geq 0$

(m) $|4 - 3x| < -2$

(n) $|5 - 3x| \leq 0$

(o) $\left| \frac{x-3}{x-5} \right| > 1$

(p) $\left| \frac{x+4}{x+2} \right| \leq 1$

(q) $|x^2 - 4x| < 5$

(r) $|x - 3| > -1$

(s) $|3x - 2.5| \geq 2$

(t) $|x - 2| \leq |x + 4|$

(a) $(-\infty, -2) \cup (2, \infty)$

(b) $(-\infty, -2) \cup (4, \infty)$

(c) $x \in (1, 3)$

(d) $x \in (-\infty, -3] \cup [1, \infty)$

(e) $x \in (-4, 6)$

(f) $(-\infty, -2) \cup (5, \infty)$

(g) $\left(-\frac{7}{3}, -1\right)$

(h) $x \in \left(-\infty, -\frac{11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

(i) $x \in (-\infty, \infty)$

(j) $x \in \left[\frac{1}{2}, 4\right]$

(k) $x \in \left(-\infty, -\frac{7}{3}\right] \cup [-1, \infty)$

(l) $x \in \mathbb{R}$

(m) $x \in \phi$

(n) $x \in \left\{\frac{5}{3}\right\}$

(o) $(4, \infty) - \{5\}$

(p) $(-\infty, -3]$

(q) $(-1, 5)$

(r) $x \in (-\infty, \infty)$

(s) $x \in \left(-\infty, \frac{1}{6}\right] \cup \left[\frac{3}{2}, \infty\right)$

(t) $x \in [-1, \infty)$

Solve the following equations

(a) $|x - 1| + |x - 3| = 2$

(b) $|x| + |x + 5| = 5$

(c) $|x - 1| + |x - 4| = 2$

(d) $|x^2 - 2x| + |x - 4| = |x^2 - 3x + 4|$

Answers:

(a) $[1, 3]$

(b) $[-5, 0]$

(c) $\{\}$

(d) $x \in (-\infty, 0] \cup [2, 4]$

B.P.P.

$$(a) |x| > 2 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$$

$$(b) |x-1| > 3 \Rightarrow x-1 < -3 \text{ or } x-1 > 3 \\ x < -2 \text{ or } x > 4 \\ \Rightarrow x \in (-\infty, -2) \cup (4, \infty)$$

$$(c) |x-2| < 1 \Rightarrow -1 < x-2 < 1 \\ 1 < x < 3 \Rightarrow x \in (1, 3)$$

$$(d) |x+1| \geq 2 \Rightarrow x+1 \leq -2 \text{ or } x+1 \geq 2 \\ x \leq -3 \text{ or } x \geq 1 \\ \Rightarrow x \in (-\infty, -3] \cup [1, \infty)$$

$$(e) |x-1| \leq 5 \Rightarrow -5 \leq x-1 \leq 5 \\ -4 \leq x \leq 6 \Rightarrow x \in [-4, 6]$$

$$(f) |2x-3| > 7 \Rightarrow 2x-3 < -7 \text{ or } 2x-3 > 7 \\ 2x < -4 \text{ or } 2x > 10 \\ x < -2 \text{ or } x > 5 \\ \Rightarrow x \in (-\infty, -2) \cup (5, \infty)$$

$$(g) |3x+5| < 2 \Rightarrow -2 < 3x+5 < 2 \\ -7 < 3x < -3 \\ -\frac{7}{3} < x < -1 \Rightarrow x \in \left(-\frac{7}{3}, -1\right)$$

$$(h) |4x+6| > 5 \Rightarrow 4x+6 < -5 \text{ or } 4x+6 > 5 \\ 4x < -11 \quad 4x > -1 \\ x < -\frac{11}{4} \quad x > -\frac{1}{4} \\ \Rightarrow x \in \left(-\infty, -\frac{11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$$

$$(i) |2x-3| > -2 \Rightarrow x \in \mathbb{R}$$

$$(j) |4x-9| \leq 7 \Rightarrow -7 \leq 4x-9 \leq 7 \\ 2 \leq 4x \leq 16 \\ \frac{1}{2} \leq x \leq 4 \Rightarrow x \in \left[\frac{1}{2}, 4\right]$$

$$(k) |2x+5| \geq 2 \Rightarrow 2x+5 \leq -2 \text{ or } 2x+5 \geq 2 \\ 2x \leq -7 \quad 2x \geq -3 \\ x \leq -\frac{7}{2} \quad x \geq -\frac{3}{2} \\ \Rightarrow x \in \left(-\infty, -\frac{7}{2}\right] \cup \left[-\frac{3}{2}, \infty\right)$$

$$(l) |2x+3| \geq 0 \Rightarrow x \in \mathbb{R}$$

$$(m) |4-3x| < -2 \Rightarrow x \in \emptyset$$

$$(n) |5-3x| \leq 0 \Rightarrow |5-3x| = 0 \\ 5-3x = 0 \\ 3x = 5 \Rightarrow x = \frac{5}{3}$$

$$(o) \left|\frac{x-3}{x-5}\right| > 1 \Rightarrow \frac{x-3}{x-5} < -1 \text{ or } \frac{x-3}{x-5} > 1 \\ \frac{x-3}{x-5} + 1 < 0 \quad \frac{x-3}{x-5} - 1 > 0$$



$$\frac{x-3+x-5}{x-5} < 0 \quad \text{or} \quad \frac{x-3-x+5}{x-5} > 0$$

$$\frac{2x-8}{x-5} < 0 \quad \text{or} \quad \frac{2}{x-5} > 0$$

$$\frac{x-4}{x-5} < 0 \quad \text{or} \quad \frac{1}{x-5} > 0$$

$$x \in (4, 5) \quad \text{or} \quad x-5 > 0 \Rightarrow x > 5$$

$$\Rightarrow \boxed{x \in (4, 5) \cup (5, \infty)}$$

$$(p) \quad \left| \frac{x+4}{x+2} \right| \leq 1 \Rightarrow -1 \leq \frac{x+4}{x+2} \leq 1$$

$$\frac{x+4}{x+2} \geq -1 \quad \& \quad \frac{x+4}{x+2} \leq 1$$

$$\frac{x+4}{x+2} + 1 \geq 0 \quad \& \quad \frac{x+4}{x+2} - 1 \leq 0$$

$$\frac{2x+6}{x+2} \geq 0 \quad \& \quad \frac{x+4-x-2}{x+2} \leq 0$$

$$\frac{x+3}{x+2} \geq 0 \quad \& \quad \frac{2}{x+2} \leq 0$$

$$x \in (-\infty, -3] \cup [2, \infty) \quad \& \quad x+2 < 0 \Rightarrow x < -2$$

$$\Downarrow$$

$$\boxed{x \in (-\infty, -3]} \quad \text{Ans}$$

$$(q) \quad |x^2 - 4x| < 5 \Rightarrow -5 < x^2 - 4x < 5$$

$$x^2 - 4x + 5 > 0 \quad \& \quad x^2 - 4x - 5 < 0$$

$$(x-5)(x+1) < 0 \quad \& \quad x \in \mathbb{R}$$

$$x \in (-1, 5)$$

$$\Downarrow$$

$$\boxed{x \in (-1, 5)} \quad \text{Ans}$$

$$(r) \quad |x-3| > -1 \Rightarrow \boxed{x \in \mathbb{R}}$$

$$(d) \quad |3x-2.5| \geq 2 \Rightarrow 3x-2.5 \leq -2 \quad \text{or} \quad 3x-2.5 \geq 2$$

$$3x \leq 0.5 \quad 3x \geq 4.5$$

$$x \leq \frac{0.5}{3} \quad x \geq \frac{4.5}{3} \Rightarrow x \geq 1.5$$

$$\Rightarrow \boxed{x \in (-\infty, \frac{0.5}{3}] \cup [1.5, \infty)}$$

$$(f) \quad |x-2| \leq |x+4| \Rightarrow (x-2)^2 - (x+4)^2 \leq 0$$

$$(x-2+x+4) \cdot (x-2-x-4) \leq 0$$

$$(2x+2) \cdot (-6) \leq 0$$

$$x+1 \geq 0$$

$$x \geq -1 \Rightarrow \boxed{x \in [-1, \infty)}$$

Q) Solve the following questions:

$$(a) \quad |x-1| + |x-3| = 2$$

$$|a| + |b| = |a-b|$$

$$\Rightarrow (x-1) \cdot (x-3) \leq 0$$

$$ab \leq 0 \Rightarrow \boxed{x \in [1, 3]} \quad \text{Ans}$$

$$(b) \quad |x| + |x+5| = 5$$

$$|a| + |b| = |a-b| \Leftrightarrow ab \leq 0$$

$$\Rightarrow x \cdot (x+5) \leq 0 \Rightarrow \boxed{x \in [-5, 0]} \quad \text{Ans}$$

$$(c) \quad |x-1| + |x-4| = 2$$

$$\text{Ans: } x \in \emptyset$$

$$(i) \text{ If } x \leq 1, \quad (ii) \text{ If } 1 < x < 4, \quad (iii) \text{ If } x \geq 4$$

$$-x+1-x+4=2 \quad x-1-x+4=2 \quad x-1+x-4=2$$

$$3=2x \quad 3=2 \quad 2x=7$$

$$x=3/2 \quad (Not \text{ possible}) \quad x=7/2$$

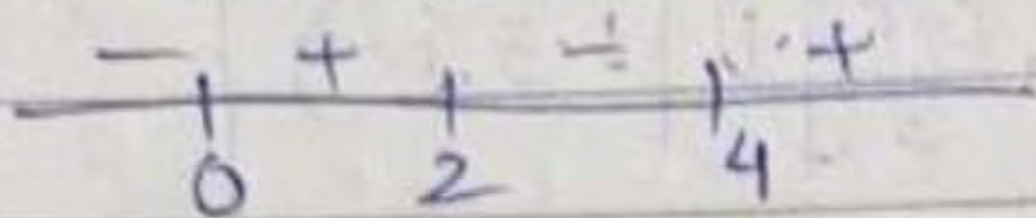
$$(reject) \quad x \in \emptyset \quad (reject)$$

Date: _____

$$(d) \quad |x^2 - 2x| + |x - 4| = |x^2 - 3x + 4|$$
$$|a| + |b| = |a - b| \iff ab \leq 0$$

$$(x^2 - 2x) \cdot (x - 4) \leq 0$$

$$x(x - 2)(x - 4) \leq 0$$



$$\Rightarrow x \in [0, 2] \cup [4, \infty)$$

$$\boxed{x \in (-\infty, 0] \cup [2, 4]}$$

Ans.

THANK
YOU